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ON THE SCATTERING OF X-RAYS BY HYDROGEN

By G. A. Schott

Abstract

Scattering of x-rays by hydrogen; comparison of experimental results with the various theories.—Measurements of the absorption and scattering of x-rays by hydrocarbon compounds made recently by Aurén, by Hewlett, and by Olsen, Dershem and Storch, are considered critically insofar as they serve for the calculation of the mass-scattering coefficient of hydrogen for different wave-lengths and its comparison with the results computed on the basis of (1) the Simple Pulse theory, (2) the Electron Ring theory, (3) the Ring-electron theory and (4) the Quantum theory of scattering recently proposed by A. H. Compton. It is found that none of these theories, even when it is interpreted in the widest sense, agrees with the experimental results over the whole range of wave-lengths. This failure may very well be due to the inaccuracy in the experimental results since the method of calculating the mass-scattering coefficient from measurements made upon compounds necessarily magnifies the experimental errors very considerably owing to the smallness of the scattering of hydrogen as compared with that of the remaining constituents. The importance of obtaining reliable data is pointed out and it is suggested that sufficiently accurate results can be secured only by using the element in the free state, either in the liquid form, or as gas under high pressure.

RECENT measurements of the absorption and scattering of x-rays have been made by Aurén,[1] Hewlett,[2] and Olson, Dershem and Storch.[3] In his second paper Aurén determined the total absorption of nearly homogeneous x-rays by 13 hydrocarbon derivatives for 6 wave-lengths from 0.359 A to 0.194 A. Hewlett measured directly the scattering of the $K\alpha$ line of molybdenum by carbon, lithium, benzene, mesitylene and octane for a range of angles from 5° to 175° and deduced the total scattering. Finally Olson, Dershem and Storch measured the total absorption of homogeneous x-rays by 7 hydrocarbon derivatives for wave-lengths from 0.1899 A to 0.9773 A. On the assumption that the

[1] T. E. Aurén, Phil. Mag. **37**, p. 165, 1919; Meddelanden fr. Nobelinstitut, **4**, No. 5, 1920

[2] C. W. Hewlett, Phys. Rev. **20**, 688, 1922

[3] A. R. Olson, Elmer Dershem and H. H. Storch, Phys. Rev. **21**, 30, 1923

additive law holds for the absorption and scattering by all the substances studied for each wave-length—an assumption which is verified by the experimental results themselves—these authors have calculated the mass-scattering coefficients of H, C and O for a number of wave-lengths on the further assumption that the absorption is entirely due to scattering. This further assumption is probably true for these light elements, at any rate for hydrogen, with which we are more particularly concerned, since there can hardly be any resonance for wave-lengths as small as those used in these experiments. We shall confine our discussion to the case of hydrogen because the theoretical constitutions proposed for its atom and molecule are so simple that the scattering coefficients can be calculated with sufficient certainty and precision according to the various theories of scattering that have been proposed.

When we examine the values of the total mass-scattering coefficient of hydrogen employed, we are struck at once by the largeness of the experimental errors with which they are affected, especially as regards the results of Olson, Dershem and Storch; but even for Aurén's experiments the probable error (of the order 5 per cent) is too large to allow us to decide definitely between the various theories. I wish to lay special stress upon the fact that this difficulty arises primarily from the method of calculation, not from undue inaccuracy of measurement, for clearly all reasonable precautions have been taken to minimize errors in the actual experiments. The scattering of hydrogen is so small in comparison with that of the remaining constituents of the compounds used that the unavoidable experimental errors are magnified enormously in the calculation of the scattering coefficient of hydrogen. For instance Aurén calculated his latest results by the method of least squares; in a private communication, for which I am much indebted to him, he gives the probable errors of the scattering coefficient of hydrogen, referred to water as a standard. They range from 2.68 to 5.76 per cent; this is somewhat less than the deviations of the scattering coefficient from J. J. Thomson's value 0.4, but it is so little less that the deviations can hardly be regarded as definitely established. It might be argued that the number of measurements is not large enough to justify the use of the method of least squares; nevertheless this method enables us to avoid all arbitrariness, in so far as all the observations are treated exactly alike. Olson, Dershem and Storch, like Aurén in his first paper, select the combinations of substances used in their calculation a little arbitrarily; moreover they do not use their actual results, but values obtained from somewhat arbitrarily smoothed out graphs of the absorption coefficients of the compounds examined. The smoothing out tends to remove undulations in the curves

which represent the absorption coefficients as functions of the wave-length; it treats these undulations as due to errors of measurement, whereas they may have a real existence, at least according to some theories of scattering.

For these reasons I have thought it worth while to recalculate both Aurén's values and those of Olson, Dershem and Storch using the method of least squares. The results of this recalculation, as well as those given by Aurén and by Olson, Dershem and Storch themselves are given in Table I. The numbers given in column 1 are the wave-lengths in Angström units; those in column 2 the mass-scattering coefficients of hydrogen given by Olson, Dershem and Storch in their Table III; in column 3 the same recalculated by myself from their Table I; in column 4 Aurén's values from his Table V; and in column 5 the values recalculated by myself from Aurén's by means of his Table IV and his formula (5).

TABLE I

	Results of Olson, Dershem and Storch		Results of Aurén	
λ	As given	Recalculated	As given	Recalculated
0.194 A			0.340	0.338
.2137		0.339		
.215			.349	.338
.237			.356	.351
.2455	.382	.333		
.264			.362	.360
.2782	.407	.531		
.302			.369	.377
.3109	.445	.420		
.3436	.425	.449		
.359			.375	.410
.3763	.441	.465		
.4089	.462	.385		
.4416	.442	.517		
.4905	.446	.459		
.5394	.452	.412		
.5883	.439	.333		
.6371	.435	.349		
.6859	.441	.489		
.7346	.482	.445		
.7833	.499	.450		
.8319	.532	.454		
.8804	.553	.637		
.9289	.589	.325		
.9773	.617	.454		

No doubt the differences between columns 2 and 3 arise, partly at least, from Olson, Dershem and Storch's method of a preliminary smoothing out of the experimental curves. The small but gradually increasing differences between columns 4 and 5 are due to the fact that Aurén in his calculations approximates by omitting the small term $29q$ in the

denominator of his Eq. (5), and treating the quantity $q(\mu/\rho)_{Cu}$ as constant. I have preferred to avoid these approximations.

It should be added that Hewlett's value for hydrogen, 0.463 for $\lambda=0.712$ A agrees very well with the numbers in columns 2 and 3. All these results are plotted in Fig. 1.

In addition to the numbers given in Table I, Olson, Dershem and Storch give a second series of results, but I have not thought it worth while to include them, for the number of compounds examined is smaller, and the range of wave-lengths much smaller than it is for the first set of experiments, so that the accuracy of the results will be much less, while the labor of calculation is very great. Besides, the experimental values agree very well with those obtained in the first set; moreover Olson, Dershem and Storch themselves apparently make no use of them in calculating their final Tables II and III.

In order to compare the experimental values of the mass-scattering coefficient of hydrogen for different wave-lengths with the theoretical values we need to consider four theories: (1) the Simple Pulse theory of J. J. Thomson;[4] (2) the Electron Ring theory;[4] (3) the Ring-electron theory;[4] and (4) the Quantum theory recently proposed by A. H. Compton.[5] We shall not consider the theory, also due to A. H. Compton, which is based on the assumption that the electron is a flexible sphere of radius about 10^{-10} cm because the electromagnetic mass of so large a sphere would be only about one-thousandth of the actual mass of the negative electron, which is a serious theoretical objection to the theory.

We shall now summarize the formulas for the mass-scattering coefficient of hydrogen given by these four theories.

(1) The Simple Pulse theory of J. J. Thomson gives

$$\sigma=8\pi e^4/3c^4m^2M,$$

where e denotes the charge of the negative electron, $4.774(10)^{-10}$ e.s.u. according to Millikan, e/cm its specific charge, $1.765(10)^7$ e.m.u. according to Neumann and other recent experimenters, c the velocity of light, $2.999(10)^{10}$ cm/sec., and M the mass of the hydrogen atom, so that $e/cM=9647$ e.m.u. With these values we obtain $\sigma=0.401$.

It should be noticed that on this theory we obtain exactly the same value of σ for molecular as for atomic hydrogen, because for the former we must reckon two electrons per molecule, but must double M at the same time. It is clear from the numbers given in Table I that this theory is quite untenable, for experiment shows that the mass-scattering coefficient

[4] For a discussion of these theories cf. G. A. Schott, Proc. Roy. Soc. A **96,** 395, (1920)
[5] A. H. Compton, Phys. Rev. **21,** 493 (1923)

of hydrogen for short waves is 15 per cent less, and for long waves from 15 to 50 per cent greater than the value just calculated.

(2) The Electron Ring theory gives $\sigma = 0.401$ just like the Simple Pulse theory, if we make the natural supposition that hydrogen in hydrocarbon compounds is atomic, with a single electron in the ring assumed circular; thus this view is negatived by experiment, at any rate so far as the results detailed here are reliable. It is the view adopted in most theories including that of Bohr; what the result would be for an elliptic orbit is not known. If on the other hand we assume that the hydrogen is molecular—a view obviously difficult to sustain on other grounds in such a compound as toluene with a single ring of two electrons at opposite ends of a diameter, we find[6]

$$\sigma = \sigma_0\{1+f(r)\},$$

where $\sigma_0 = 8\pi e^4/3c^4m^2M$, $r = 4\pi\rho/\lambda$, $f(r) = \{S_0^2(r)+\frac{1}{2}S_2^2(r)\}r^{-2}$,

$$S_0(r) = \sin r,\ S_2(r) = (3r^{-2}-1)\sin r - 3r^{-1}\cos r$$

while ρ is the radius of the common orbit, supposed circular, and λ is the wave-length of the incident radiation. The values of $f(r)$ have been tabulated on p. 423 and graphed in Fig. 2 of my first paper.[4] The scale of the graph for molecular hydrogen is twice that for helium, so that the least value of σ is 0.401 for $\lambda = 0$ and the greatest 0.802 for $\lambda = \infty$, resonance being neglected.

It is clear that this view also fails, for experiment indicates that the least value for σ is not greater than 0.34; but it is superior to the previous view, for it allows sufficiently for the great increase in the value of σ shown by all the experiments for large values of λ.

If however we modify the theory by assuming that for some unknown reason the effective mass of the electrons in their circular orbit is greater than that of a free slowly moving electron in the ratio 13:12, we can take $\sigma_0 = 0.340$ and so remove the first difficulty.

The theoretical graph for the function $f(r)$ shows minima for $r = 3.1, 6.1 \ldots$, and maxima for $r = 4.2, 7.6 \ldots$ but only the first minimum and first maximum are appreciable. The experimental values of σ plotted in Fig. 1, at least those of Olson, Dershem and Storch, give indications of a minimum near $1/\lambda = 1.6$, and of a maximum near $1/\lambda = 2.1$ or 2.2. If we take $4\pi\rho = 2$, or $\rho = 0.16$ A, the first theoretical minimum and maximum fall nearly in their observed positions. The theoretical curve for σ constructed with $\sigma_0 = 0.340$ and $\rho = 0.16$ A is shown as curve 1, 1, and that with $\sigma_0 = 0.401$ and the same value of ρ, as curve 2, 2 in Fig. 1.

[6] Schott, l.c.,[4] p. 408, Eqs. (27) and (29)

(3) The Ring-electron theory gives[7]

$$\sigma=\sigma_0\sum_0^\infty (-1)^k\frac{3(k^2+3k+4)r^{2k}}{2(\underline{|k})\ (\underline{|k+3})\ (2k+1)}$$

$$=\frac{11\sigma_0}{5r}\left\{\tfrac{1}{2}\int_0^{2r}J_0(x)dx-\tfrac{1}{2}J_1(2r)+\frac{J_2(2r)}{11r}-\frac{6J_3(2r)}{11r^2}\right\}$$

$$=\frac{11\sigma_0}{5r}\left\{\tfrac{1}{2}J_1(2r)+\frac{J_2(2r)}{11r}+(1-\frac{6}{11r^2})J_3(2r)+J_5(2r)+J_7(2r)+\ \ldots\right\}$$

The series converges, but only very slowly for large values of r; the four first terms suffice to $r=1$ for an accuracy of 1 per mille, which is well beyond the experimental accuracy. The second expression is easily derived from Eq. (42) of my first paper[4] by direct integration, but is more easily verified by expansion and comparison with the series. The third expression is obtained at once by the substitution of the well known Bessel function series for the Bessel function integral; it is convenient for computation for half-integral values of r greater than unity, for which values tables exist. By these means Table II has been computed.

TABLE II

Values of σ/σ_0

r:	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.5
σ/σ_0:	.998	.993	.985	.974	.960	.943	.923	.900	.876	.850	.702
r:	2.0	2.5	3.0	3.5	4.0	4.5	5.0	6.0	8.0	10.	
σ/σ_0:	.554	.432	.350	.303	.272	.250	.227	.181	.140	.109	

If we put for σ_0 its theoretical value, which is the same as the Pulse theory value 0.401, we cannot secure a satisfactory agreement with the experimental values for long waves, for these exceed 0.401 very considerably. If however we assume that the effective mass of the ring-electron in the atom is less than that of the free electron for some unknown reason and take $\sigma_0=0.520$ and $\rho=0.0276$ A (which makes $r=0.35$ correspond to $\lambda=1$ A), we can secure a better agreement. With these values I have plotted the curve 3.3 in Fig. 1.

(4) The Quantum theory gives[8] $\sigma=\sigma_0/(1+2\alpha)$, where $\alpha=h/cm\lambda=0.02420/\lambda$, h being Planck's constant. It is to be noted that the expression here used for σ is what Compton calls the scattering absorption coefficient and includes not only the true scattering but also an absorption due to transference of energy from the radiation to the recoiling electron. The experimentally determined value of σ however includes losses from all sources and therefore must be identified with the scattering absorption coefficient as above.

[7] Schott, l.c.,[4] p. 417, Eq. (44)

[8] A. H. Compton, l.c.,[5] p. 493, Eq. (28)

The curve which gives σ as a function of $1/\lambda$ is a rectangular hyperbola whose asymptotes are the axis of λ and the line $1/\lambda = -20.66$, or $\lambda = -0.0484$. Thus $\sigma = \sigma_0$ when $\lambda = \infty$, and $\sigma = 0$ when $\lambda = 0$.

Compton identifies σ_0 with the Simple Pulse theory value, $8\pi e^4/3c^4m^2M$, or 0.401, but clearly this value makes it impossible to account for the excess scattering at large wave-lengths. It does not seem to me that

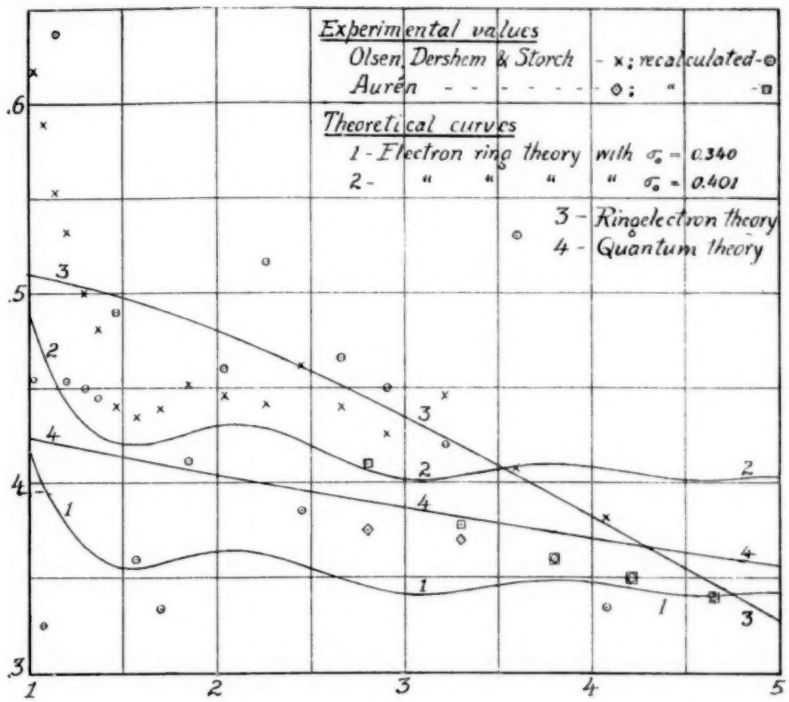


Fig. 1. Atomic scattering coefficient of hydrogen, as a function of $1/\lambda$. For curve 3, $\sigma_0 = 0.520$ and $\rho_0 = 0.0276$ A; for curve 4, $\sigma_0 = 0.443$.

the Quantum theory in itself assigns any definite value to σ_0 at all, so that there is apparently no vital reason why σ_0 should not be regarded as a disposable constant. At any rate if we treat it as arbitrary and α as determinate, we find by the method of least squares that $\sigma_0 = 0.443$. The corresponding curve is shown in Fig. 1 and marked 4,4.

A study of Fig. 1 shows that not one of the theoretical curves fits all the experimental results satisfactorily over the whole range of wave-lengths. The strict Electron Ring theory curve 2,2 represents the wavy curve of Olson, Dershem and Storch qualitatively, but not quantitatively; in the former the waves are not sufficiently pronounced, nor does the curve rise sufficiently rapidly for large wave-lengths; moreover it does not fit Aurén's values at all, because its limit for short waves, 0.401, is

15 per cent above Aurén's smallest values. On the other hand the modified Electron Ring theory curve 1,1 fits Aurén's values for short waves well, but falls very far below the values for long waves. We could improve it in this respect by a different choice of the radius ρ of the orbit, e.g. $\rho = 0.1$ A or so instead of 0.16 A, but the effect of such a change would be to shift the minimum and maximum to wave-lengths smaller in proportion to ρ. Neither value of ρ however agrees with Bohr's model, which requires $\rho = 0.56$ A for the hydrogen molecule, so that the curve is unsatisfactory from the theoretical point of view also.

On the other hand neither the Ring-electron theory curve 3,3 nor the Quantum theory curve 4,4 gives any indication of waves at all; the former is convex upwards, and the latter convex downwards throughout the whole range of wave-lengths under consideration. According to the Ring-electron theory the value of σ appears to diminish faster for short waves than Aurén's values indicate, but it fits the experimental values fairly well on the average in the middle of the range. We must however bear in mind that this fairly good fit is only secured by choosing for σ_0 a value, 0.520, which is 30 per cent greater than the theoretical one and requires a mass of the electron which is 15 per cent less than that found by experiment for slowly moving free electrons. So small a mass is very difficult to justify theoretically. The Quantum theory curve 4,4 fits the experimental results rather better on the whole than the curve 3,3, especially at the smaller wave-lengths, though it is inferior to it for long waves. In this case too the value of σ_0 selected is not that given by theory, but I do not regard this as a serious objection for the reason already given above.

To sum up the results of the discussion of this section we may say that the experimental results, such as they are, appear to give a wavy curve with a minimum and a maximum, showing on the whole a diminution of the mass-scattering coefficient σ with diminishing wave-length λ from about 0.5 at 1 A to about 0.34 at 0.2 A. J. J. Thomson's Simple Pulse theory will not account for this behavior at all, nor will the Electron Ring theory do so, unless the hydrogen be assumed to be present in the molecular form in the compounds examined; even then it will not do so quantitatively for the whole range of wave-lengths, because the variations which it gives for σ are not sufficiently pronounced. On the other hand the Ring-electron theory and the Quantum theory will not account for the wavy character of the experimental curve, but by a suitable choice of the quantity σ_0 they can be made to fit the experimental curve fairly well, the Ring-electron theory better at large and the Quantum

theory better at small wave-lengths. None of the theories however seems to be quite satisfactory.

In view of the want of agreement between the experimental results and the various theoretical curves even on a liberal interpretation of the theories the question arises whether after all the fault does not lie with the experimental results. It is essential that the probable error of the value of σ calculated from the measurements be reduced considerably below the value reached in these experiments—about 5 per cent for Aurén's, probably greater for those of Olson, Dershem and Storch, to judge from the want of agreement between their values and the recalculated ones given in Table I. It is hardly likely that the method of measurement can be improved sufficiently for this purpose; the only feasible way appears to be by a more suitable choice of material. From the point of view of theory hydrogen is much the best, perhaps the only substance to be used; the constitution of other substances is too complicated to afford theoretical results fit for comparison with experiment. If the magnification of errors due to calculation from results of measurements on compounds of hydrogen is to be avoided—an essential precaution—experiments must be made on hydrogen itself, preferably in the liquid form and purified to the highest degree possible. In the letter from Aurén already referred to he gives it as his opinion that such experiments would not be difficult to carry out with the full resources of a cryogenic laboratory by an experimenter versed in the technique of measurements on the scattering of x- and γ rays. The experiments should give definite answers to three questions: (1) Are the waves seen in Fig. 1 real, or are they due to experimental errors magnified by calculation? (2) Is the great excess of scattering above the Simple Pulse theory's value shown in Fig. 1 for long waves real? (3) Is the defect shown for short waves real?

UNIVERSITY COLLEGE OF WALES,
ABERYSTWYTH,
September 7, 1923.

THE INTENSITY OF THE X-RAYS SCATTERED FROM ROCKSALT

By G. E. M. Jauncey and H. L. May

Abstract

Absolute intensity of x-rays scattered from a single rock-salt crystal.— Rays from a Mo target (47.5 kv) filtered through a zirconium phosphate screen were directed onto a thin slab of rock salt at an angle θ with the normal to the face, and the scattered intensity after transmission was measured with a long ionization chamber at various angles φ with incident radiation. Assuming the scattering independent of θ and eliminating μ by using Crowther's expression for scattering coefficient when $\theta=\frac{1}{2}\varphi$, scattering in excess of theoretical (Thomson) was observed for φ between 9° and 90°, the two maxima at 15° and 30°, about twice theoretical, corresponding respectively to maxima in the incident radiation at about .4 and .7 A, respectively. Similar *excess scattering* at 20° to 30° had previously been reported for amorphous carbon and liquids but in these cases the excess scattering might have been due to a powdered crystal effect. The scattering drops suddenly to zero at 5°, about the same angle as that for which regular reflection becomes zero. Above 90°, the scattering also departs somewhat from the Thomson curve, showing a minimum for about 100°; it corresponds in shape better with the curve given by the quantum theories of A. H. Compton and Jauncey, the experimental value being, however, greater in the ratio 9/8. The *mass scattering coefficient* for the primary rays used (.4 to .7A) comes out $0.23\pm.02$ as against Thomson's value of 0.193.

1. Introduction

EXPERIMENTS on the scattering of x-rays from crystals of rocksalt and calcite have recently been described,[1] in which were obtained curves for the scattering between 15° and 150°. These curves were compared with the scattering curve for the amorphous substance glass, and it was found that the relative scattering curves were the same for the crystalline substances as for the amorphous substance between 50° and 150°. However, the scattering curves were relative and not absolute. Furthermore the relative scattering at angles below 30° was somewhat uncertain. It was, therefore, desirable to obtain the absolute scattering curves and also to obtain the scattering at angles less than 30°. In the experiments described in the present paper the absolute scattering curve for rocksalt between 5° and 160° has been obtained.

Curve A, Fig. 1 is the curve obtained by Barkla and Ayers[2] for the scattering from amorphous carbon. Curve B is the theoretical scattering

[1] G. E. M. Jauncey, Phys. Rev. **20**, 405 (1922)
[2] Barkla and Ayers, Phil. Mag. **21**, 270 (1911)

curve according to the theory of Sir J. J. Thomson.[3] It is seen that at angles below 40° the experimental value of the scattering is greater than the theoretical value, in fact, at 20° the experimental value is double the Thomson value. This scattering in excess of the Thomson value is now called excess scattering. Recently Hewlett[4] has also obtained scattering curves for carbon and certain organic liquids. Hewlett's curve for the liquid benzene exhibits a maximum at 8°, the scattering at this angle being

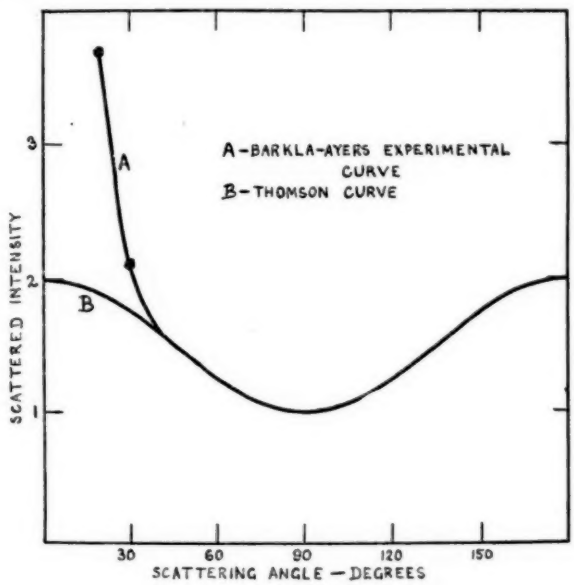


Fig. 1. Scattering from amorphous carbon compared with theoretical curve.

about 14 times the scattering at 90°. This excess scattering may, however, be due to a powdered crystal effect, even when a liquid is used.

The question then arises as to whether there is any such thing as true excess scattering. To answer this question it is necessary to remove all crystalline reflection from the scattered beam. This can be done if a large single crystal is used, for then the regular reflection can occur only at certain angles.[1]

2. Experimental Methods

The x-rays from a Coolidge tube with a molybdenum target were allowed to fall upon a crystal of rocksalt which was mounted on the axis of a Bragg spectrometer as shown in Fig. 2. The orientation of the rocksalt crystal scattering the x-rays was different from that de-

[3] J. J. Thomson, Conduction of Electricity through Gases, 2nd. Ed., p. 325
[4] C. W. Hewlett, Phys. Rev. **20,** 688 (1922)

scribed in the previous paper,[1] in which a thick crystal was used and the scattering from the front face in various directions was studied. In the experiments described here, however, a thin crystal slab was used and the scattered x-rays which passed through the crystal were studied. The primary rays were incident at an angle θ to the normal to the

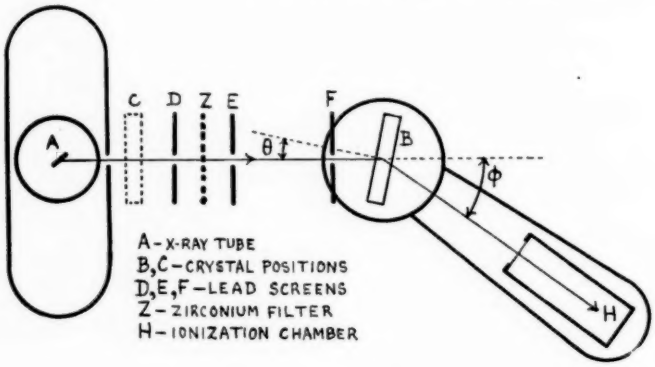


Fig. 2. Diagram of apparatus.

(1, 0, 0) face. The rays scattered in the direction ϕ entered the ionization chamber and there produced an ionization current, which was measured with a Compton electrometer operating at a sensibility of from 7,000 to 10,000 mm per volt. The relation between the electrometer deflection and the ionization current was tested and found to be linear. The distance of the window of the ionization chamber from the crystal was 14.3 cm, and the length of the ionization chamber, which was filled with air saturated with methyl-iodide vapor, was 33 cm. According to Kirkpatrick[5] this length is sufficient to absorb almost completely x-rays of the wave-lengths used. This complete absorption is necessary in measuring the intensity of scattered x-rays in different directions on account of the change in wave-length of the scattered rays with the angle of scattering, as described by Compton.[6]

The intensity of the x-rays scattered by a crystal of thickness t in a direction ϕ is given by

$$I_\phi = \frac{AsI_0 \cos(\phi-\theta)}{R^2\mu[\cos\theta-\cos(\phi-\theta)]}\left[e^{-\frac{\mu t}{\cos\theta}}-e^{-\frac{\mu t}{\cos(\phi-\theta)}}\right] \qquad (1)$$

where A is the area of the window of the ionization chamber, I_0 is the intensity of the primary x-rays incident on the rocksalt crystal, R the distance of the chamber window from the crystal, θ the angle of incidence

[5] P. Kirkpatrick, Phys. Rev. **22**, 37 (1923)
[6] A. H. Compton, Phys. Rev. **21**, 483 (1923)

of the primary rays on the crystal, μ the linear absorption coefficient of the x-rays (supposed homogeneous), and s the linear scattering coefficient per unit solid angle in the direction ϕ. From (1) it can be shown that for the case where ϕ and θ are both small the thickness t which will give the largest value of I_ϕ is equal to the reciprocal of the linear absorption coefficient. Hence crystals of thickness about 1 mm were used. The x-ray tube was excited by a Snook x-ray transformer at about 47.5 kv maximum potential difference.

It is necessary to have several lead screens D, E and F, Fig. 2, when measuring the scattering at small angles. Unless great care is taken scattering from the apertures in the lead screens is obtained. In our experiments the aperture in F, the lead screen just in front of the scattering crystal, was of such a size that it allowed the primary beam to pass through without striking the lead boundaries of the aperture, but eliminated scattering from the apertures in D and E excepting at very small angles. In spite of all these precautions there was still a small deflection of the electrometer when the crystal was removed from B. Accordingly a set of readings was taken with the crystal at C. Subtracting this latter set of readings from the set when the crystal was at B, the readings due to scattering from the crystal were obtained. The Laue spots become very numerous at small angles and it becomes desirable, therefore, to use homogeneous x-rays, but unfortunately Mo Kα x-rays obtained by reflection from a crystal give very little intensity in the scattered rays. As a compromise, the primary rays were passed through a zirconium phosphate filter at Z, Fig. 2. The spectrum of the primary rays coming through the filter is shown in curve A, Fig. 3. This was obtained by reflection from rocksalt, the angular width of the incident beam being about 30 minutes of arc. It is seen that there are two broad bands of wave-lengths with maxima at about 0.42 and 0.71 A. The filter lets through the Mo Kα line but practically removes the Mo Kβ line. In the scattering experiment the apertures in D and E were of such a size that the angular width of the primary beam was about 1°. The window of the ionization chamber was 1.5 cm high and 0.65 cm wide.

In order to find the scattering coefficient s from Eq. (1) it is necessary to know μ. However, Crowther[7] has shown that

$$I_\phi = A I s t / R^2 \cos\theta \text{ when } \theta = \phi/2, \tag{2}$$

where I is the intensity of the primary rays which penetrate through the crystal when $\theta = \phi/2$. Eq. (2) does not contain μ and the scattering coefficient s can now be calculated. However, when a crystal is used it is

[7] J. A. Crowther, Proc. Roy. Soc. **86,** 478 (1912)

just when $\theta = \phi/2$ that a Laue spot will occur at ϕ. If I_ϕ of Eq. (1) is plotted against θ, ϕ being kept constant, a curve which is very nearly a straight line is obtained as shown in the broken line in Fig. 4. It will be seen that the experimental points fall approximately on the theoretical curve except where the Laue spot occurs at $\theta = 12.5°$, ϕ being kept constant at 25°. Using this method, the Laue spots are easily distinguishable as a point corresponding to a Laue spot always falls above the straight line. Those points which fall most nearly on a straight line

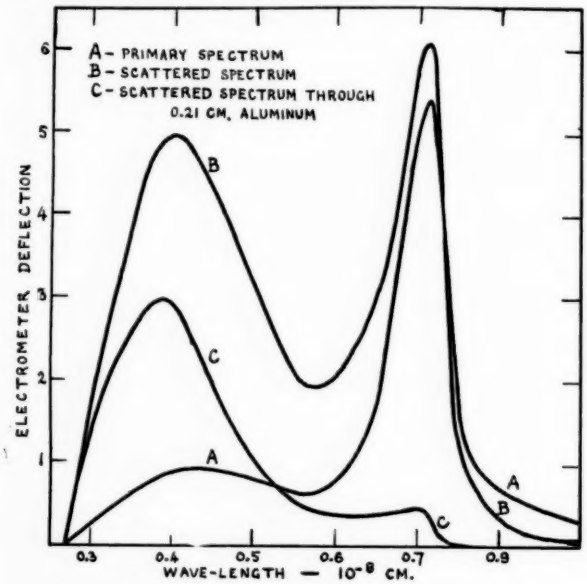


Fig. 3. Spectra of primary and scattered radiations.

are taken as being due to scattering alone. A line is drawn through them and by interpolation the scattered intensity I_ϕ is determined at $\theta = \phi/2$.

In order to determine I the chamber was set at 0° and the crystal at $\theta = \phi/2$. The electrometer deflection produced by I is many thousand times as great as that produced by I_ϕ. It is, therefore, necessary in measuring I to allow the x-rays to enter the chamber for a very short time. A pendulum arrangement was constructed in order to accomplish this. A lead sheet with an opening in it was suspended from a support by wires. When the pendulum was not vibrating the aperture in the lead sheet was directly in front of the window of the ionization chamber. If the lead sheet is pulled to one side the x-rays cannot enter the chamber. On releasing the lead sheet the aperture swings past the window and a flash of x-rays enters the chamber. The duration of the flash could be calculated and it was arranged that this duration should be 1/60

sec. This was necessary as the frequency of the alternating current operating the x-ray transformer was 60 cycles per sec. Sufficient voltage (about 70 volts) was applied to the ionization chamber to insure saturation.

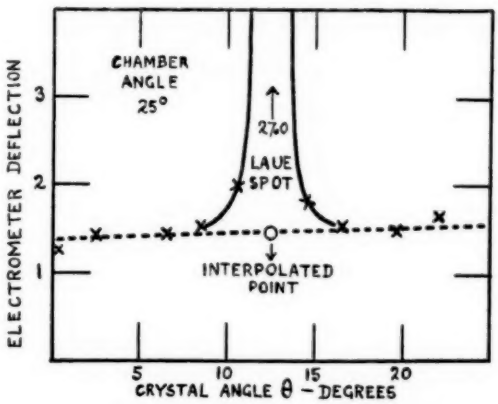


Fig. 4. Interpolation to determine I for $\theta=\frac{1}{2}\varphi$.

It might be objected that, when I is measured in the way just described, the crystal is in a position to reflect and that the increase of absorption due to reflection should be taken into account. The extinction coefficient,[8] as it is called, only becomes important, however, when reflection of a spectrum line takes place, and the Laue spots encountered in our experiments are in general not spectrum lines. Then too, the extinction coefficient is small for imperfect crystals and Davis and Stempel[9] have shown that rocksalt is a very imperfect crystal. Hence this objection to our method of measuring I has very little weight.

The experimental curves obtained are of the type shown in Fig. 4. From these, knowing I for each value of ϕ it is possible to determine s. According to Thomson's theory s should be given by

$$s=(Npd/W)\ (e^4/m^2c^4)\ \tfrac{1}{2}(1+\cos^2\phi) \qquad (3)$$

where N is Avogadro's number, p is the number of electrons per molecule, d the density of rocksalt, W its molecular weight, and e, m, and c have their usual significance. The value of s obtained from (3) when $\phi=90°$, is taken as the unit of scattered intensity. The solid curve of Fig. 5 is the experimental curve for s in these units against ϕ. The broken curve is the theoretical Thomson curve. The circles are experimental points determined absolutely by means of Eq. (2), when a crystal of thickness 0.98 mm was used. The crosses are experimental points representing

[8] Cf. Bragg, James and Bosanquet, Phil. Mag. **42**, 1 (1921)
[9] Davis and Stempel, Phys. Rev. **19**, 504 (1922)

the scattering relative to the scattering at 90°, obtained by the method described in the previous paper.

3. Discussion of Results

It is seen that the scattering curve Fig. 5 has two maxima, one at 30° and the other at 15°. The spectrum of the scattered x-rays differs from that of the primary x-rays due to the variation of the absorption coefficient with the wave-length as shown by Hewlett.[10] Using the relation that the atomic absorption coefficient of an element for a given

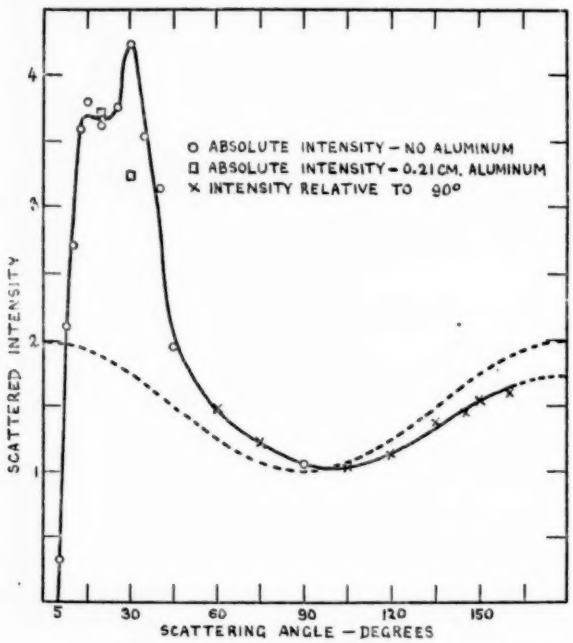


Fig. 5. Scattering at various angles.

wave-length varies with the fourth power of the atomic number, the absorption coefficient μ for rocksalt can be calculated from Hewlett's experimental values for the absorption coefficient of x-rays of various wave-lengths in aluminum. Using Eq. (1) and assuming that the scattering coefficient s is independent of the wave-length, the spectrum scattered by a rocksalt crystal of thickness 0.98 mm at $\phi = 2\theta = 30°$ is calculated from the primary spectrum shown in curve A, Fig. 3. This spectrum of the scattered x-rays is shown in curve B, Fig. 3. The ordinates of curves A and B are not on the same scale. Curve B shows two very distinct maxima at 0.40 and 0.71 A. It would seem, therefore, that even if there

[10] C. W. Hewlett, Phys. Rev. **17**, 284 (1921)

is a dependence of the scattering coefficient upon the wave-length there would still be two distinct maxima in the spectrum of the scattered x-rays. These two maxima in the spectrum might explain the two maxima in the scattering curve of Fig. 5. To test this, a thickness of 0.21 cm of aluminum was interposed in the primary beam. The effect of this was to reduce very greatly the intensity of the band of wave-lengths at about 0.71 A relatively to the intensity of the band at about 0.40 as shown in curve C of Fig. 3, which was obtained from curve B by using Hewlett's values of the absorption coefficient[10] for x-rays of different wave-lengths in aluminum. The scattering coefficients obtained at 20° and 30° using this thickness of aluminum are shown as squares in Fig. 5. It is seen that the maximum at 30° is now removed while the maximum at 15° remains. Evidently the long wave-lengths give a maximum scattering at a greater angle than the shorter wave-lengths; in particular a wave-length of 0.71 A gives a maximum at 30° and a wave-length of 0.40 A a maximum at about 15°. Comparing the spectrum curves A and B of Fig. 3, it is seen that the scattered rays are much harder than the primary rays. This is of course due to the different absorption coefficients of the different wave-lengths in rocksalt, and would not have been the case if homogeneous x-rays had been used.

The experimental scattering curve of Fig. 5 lies above the Thomson curve at angles below 90° down to 9°, and there is, therefore, a true excess scattering between these angles which is not due to a powdered crystal effect. Moreover, comparing the maxima at 15° and 30°, it would appear that the excess scattering is less for shorter wave-lengths. Below 15° there is a sudden drop in the scattering to zero at about 5°. The determination of this angle of zero scattering is difficult owing to the large area of the ionization chamber window and also the width of the primary beam which are necessary in scattering experiments. It is interesting to note from Fig. 3 that the minimum wave-length in the spectrum of the primary rays is 0.267 A. The glancing angle for this wave-length when reflected from the (1, 0, 0) face of rocksalt is 2°43′, making the total angle of deviation 5°26′. There are very few planes of greater grating space than that between the (1, 0, 0) planes in rocksalt and it seems therefore that possibly the scattering goes to zero at the same angle as the regular reflection for a given wave-length.

At angles greater than 90° the scattering falls below the Thomson value, the minimum of the curve occurring not at 90° but at about 100°. It is interesting to compare our experimental value of the scattering with the intensity predicted by the quantum theory of scattering as

developed by Compton[6] and Jauncey.[11] Taking a wave-length midway between the two maxima in curve B, Fig. 3, an average wave-length of 0.545 A is obtained. This makes the quantity $\alpha(=h/mc\lambda)$ of the above papers equal to 0.045. For this va ue of α the expressions obtained by Compton and Jauncey for the intensity of the scattering at different angles ϕ become practically identical. In Fig. 6, it is seen that from 90°

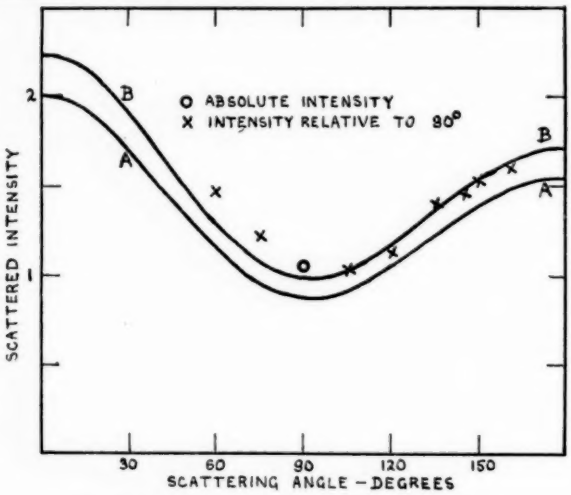


Fig. 6. Comparison with quantum theory.

to 160° the experimental points all fall above the theoretical curve A. However, if the ordinates of curve A are all multiplied by a factor 9/8 such that the new curve passes through the experimental point at 105° then the experimental points from 90° to 160° fall very closely on this curve, though below 90° the curve does not agree any better with the experimental values than does Thomson's curve.

The spherical scattering coefficient was obtained from the scattering curve of Fig. 5. Multiplying the ordinates of the experimental and Thomson curves by sin ϕ, two new curves were obtained. By comparing the areas under the curves, the ratio of the experimental spherical scattering coefficient to the Thomson coefficient was obtained. This ratio is 1.19. The mass scattering coefficient on Thomson's theory is 0.4019 (p/W), where p is the number of electrons in the molecule and W the molecular weight of the scattering substance. The mass scattering coefficient for rocksalt is therefore 0.193 on Thomson's theory. Our experimental value is 0.23. The error in these experiments is somewhat difficult to determine, but is probably between 5 and 10 per cent.

WASHINGTON UNIVERSITY,
ST. LOUIS, MO.,
June 16, 1923.

[11] G. E. M. Jauncey, Phys. Rev. **22**, 233 (1923)

DIRECTION OF EJECTION OF PHOTO-ELECTRONS BY POLARIZED X-RAYS

By Frank W. Bubb

Direction of ejection of photo-electrons by polarized x-rays.—Stereoscopic photographs were obtained, by Wilson's cloud expansion method, which show the ionized tracks of photo-electrons ejected by plane polarized x-rays. The polarized x-rays, scattered by a paraffin block at 90° to an unpolarized primary beam of hard x-rays, were directed horizontally through the expansion chamber of a Wilson cloud apparatus in which they produced the photo-electrons. Exploded tungsten wires furnished the instantaneous illumination of the droplets. The photographs, taken with the plate at 90° to the polarized beam, show two types of asymmetry in the direction of ejection of the photo-electrons. *Lateral asymmetry.* There is a strong concentration of photo-electrons ejected nearly in the direction of the electric vector of the plane polarized radiation performing the ejection. *Longitudinal asymmetry.* Stereoscopic examination of the photographs shows one-sixth of the photo-electrons ejected with a component opposite to the beam, one-third ejected approximately at right angles to the beam, and one-half ejected with a component along the beam. *Theoretical interpretation according to the classical and quantum theories.* The results are in accord with the classical theory. To explain them on the quantum theory we must assume that the quantum is a vector bundle of energy, for it explodes, so to speak, at right angles to its direction of motion.

Introduction

FROM an examination of one of C. T. R. Wilson's cloud expansion photographs[1] of the ionized tracks of secondary β-particles produced in air by x-rays, A. H. Compton notes[2] that 21 tracks start in directions within 45° of a plane normal to the beam of x-rays, 7 start at about 45° and only three start at angles greater than 45°. This indicates that most of the photo-electrons start nearly parallel to the plane of the electric and magnetic vectors of the radiation. In this connection Compton suggested that "it would be of great interest to examine by this method (Wilson's cloud method) the initial direction of the electrons ejected by polarized x-rays. It seems probable that the direction of ejection should be close to that of the electric vector."

Pohl and Pringsheim's work[3] on the selective photo-electric effect shows that a beam of plane polarized ultra-violet light produces a much larger photo-electric current from certain flat metallic surfaces when the electric force of the radiation has a component perpendicular to the surface than

[1] C. T. R. Wilson, Proc. Roy. Soc. **87,** Plate 8, No. 4, (1912)
[2] A. H. Compton, Bull. Nat. Res. Coun. No. 20, p. 25, (1922)
[3] R. Pohl and P. Pringsheim, Deutsch. Phys. Gesell. Verh. **13,** 474 (June 30, 1911)

when the electric force is tangential to the surface. The supposition that the photo-electrons are ejected nearly parallel to the electric force is in accord with this result, because surely those electrons ejected with a velocity component perpendicular to the surface have better chances of escaping than those have which are ejected tangentially to the surface.

According to the classical electromagnetic theory, the electric force is perpendicular to the direction of propagation of an electro-magnetic wave. If the electric force ejects the photo-electrons, there should then be, in the case of a plane polarized beam, a concentration of photo-electrons nearly in a single direction at right angles to the beam.

The present experiment was performed for the purpose of definitely testing whether there exists any relation between the direction of ejection of photo-electrons and the direction of the electric force of the radiation producing the photo-electrons.

Experimental Procedure

The tracks of photo-electrons ejected by polarized x-rays were photographed by Wilson's cloud method, using the apparatus shown diagrammatically in Fig. 1.

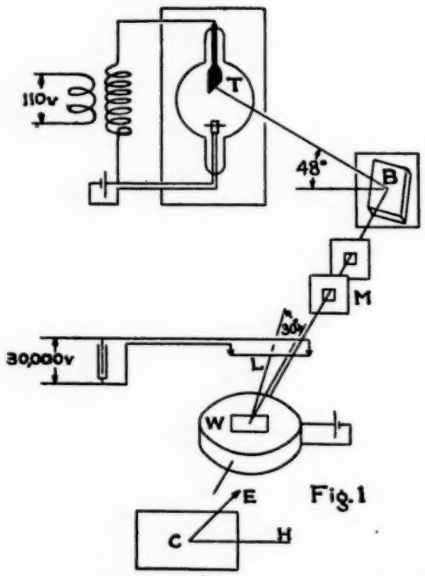


Fig. 1. Polarized x-rays from the scattering block B pass through the expansion chamber W of a Wilson cloud apparatus and eject photo-electrons nearly parallel to a line CE in the direction of the electric force of the radiation.

An unpolarized beam of x-rays TB, from the tungsten target T of a Coolidge tube driven at about 50 kv and 50 mil-amp., is directed at about 45° to the horizontal upon a paraffin scattering block B. From this

scattering block a plane polarized beam of x-rays *BC* proceeds horizontally at 90° to *TB*, through two lead collimating plates *M*, and through the expansion chamber *W* of a Wilson cloud apparatus. The center of a photographic plate *C* is placed in line with and perpendicular to this scattered x-ray. The two lines *CH* and *CE* shown on plate *C* are respectively a horizontal line and a line parallel to the electric force of the polarized beam. The angle between them was about 45°.

On the under side of the plate glass top of the expansion chamber was a transparent conducting gelatin film. On the piston top there was a conducting layer of gelatin blackened with india ink. A constant potential difference of about 60 volts per cm was maintained between the two gelatin layers for the purpose of keeping the chamber free of stray ions until the photograph was taken. The chamber was screened from all but the polarized beam *BC* by placing the Coolidge tube in a lead box, the polarization block in another lead box and by using lead collimators.

To photograph the condensed water droplets along the ionized track of the photo-electron, a very intense instantaneous source of illumination is necessary. Tungsten wires, *L*, .025 mm in diameter and 10 cm long, were stretched between *V* notches and exploded by the current from a condenser of about .04μf capacity charged to about 30 kv by a small transformer having a thermionic valve in circuit. Three wires were generally used for each explosion. Behind the wires was placed a small plane mirror to reflect more light into the chamber. The light entered the chamber through a slot in a piece of black paper covering the chamber top and was inclined at about 30° with the beam of x-rays. It illuminated a small bent wire fastened to the piston which also served as a mark for focusing the camera.

A stereoscopic camera was used. The matched lenses were marked 5.5″ focus, *F* 4.5, and were furnished by the Graf Optical Co. These were set 7.3 cm center to center on the lens board. A piece of black card board extended from the lens board to the plate for the purpose of assuring that each half of the plate was exposed to only one lens. Graflex Seed 60 plates 5″×7″ were found satisfactory.

The timing of the various events of the experiment was accomplished by means of a heavy pendulum which actuated various suitable devices placed in its path. After the room had been darkened and the camera opened, the pendulum was released, resulting in the following events. (1) A weight was released, which pulled the stopper separating the space under the piston from the vacuum chamber, thus producing the expansion. (2) The electric field across the top and bottom of the expansion chamber was removed by breaking the circuit of the battery.

(3) The x-ray tube was started by closing the primary circuit of the transformer. (4) The tungsten wires were exploded thereby illuminating the tracks.

The time intervals between these events were adjusted by trial to give the best photographs; no attempt has been made to calculate them accurately.

Experimental Results

Stereoscopic photographs of the ionization tracks of photo-electrons produced in moist air by plane polarized x-rays were obtained. Some of these photographs are shown in Plate I.

The directions of ejection of the photo-electrons were obtained by measuring the angles made by the initial portions of the tracks with a horizontal line represented by *CH* in Fig. 1. Fig. 2 shows graphically the results of these measurements.

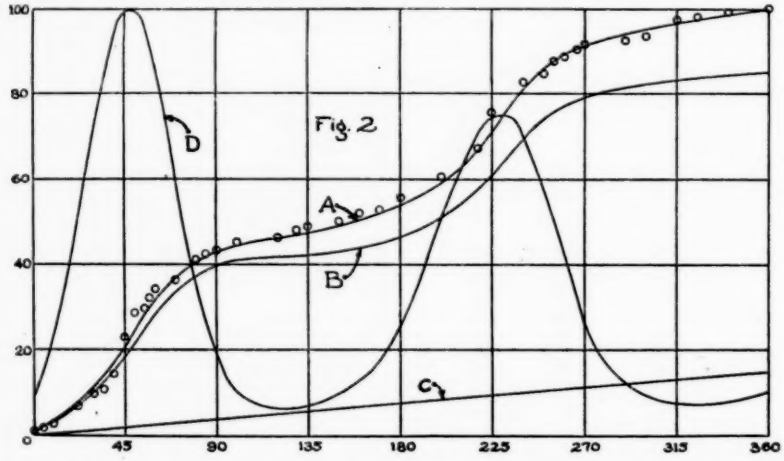


Fig. 2. Two maxima of curve D at 48° and 228° show strong concentration of photo-electrons ejected nearly parallel to electric force of polarized radiation.

In Fig. 2 the abscissas represent the angles, measured counterclockwise from *CH*, of the initial portions of the tracks. In curve A the ordinates represent the relative number of tracks whose initial directions fall between the zero line and the line fixed by the abscissa. For instance the ordinate 43 at 90° is the percentage falling between 0° and 90° of the total number of tracks and not the number starting at 90°. In plotting this curve 156 tracks were considered whose points of origin could be definitely recognized.

Curve B was obtained from curve A by allowing for the non-polarized portion of the beam of x-rays. A scattered (not fluorescent) beam at 90° with the primary is, according to the classical theory, completely polarized. However, compound scattering by thick scattering blocks and the

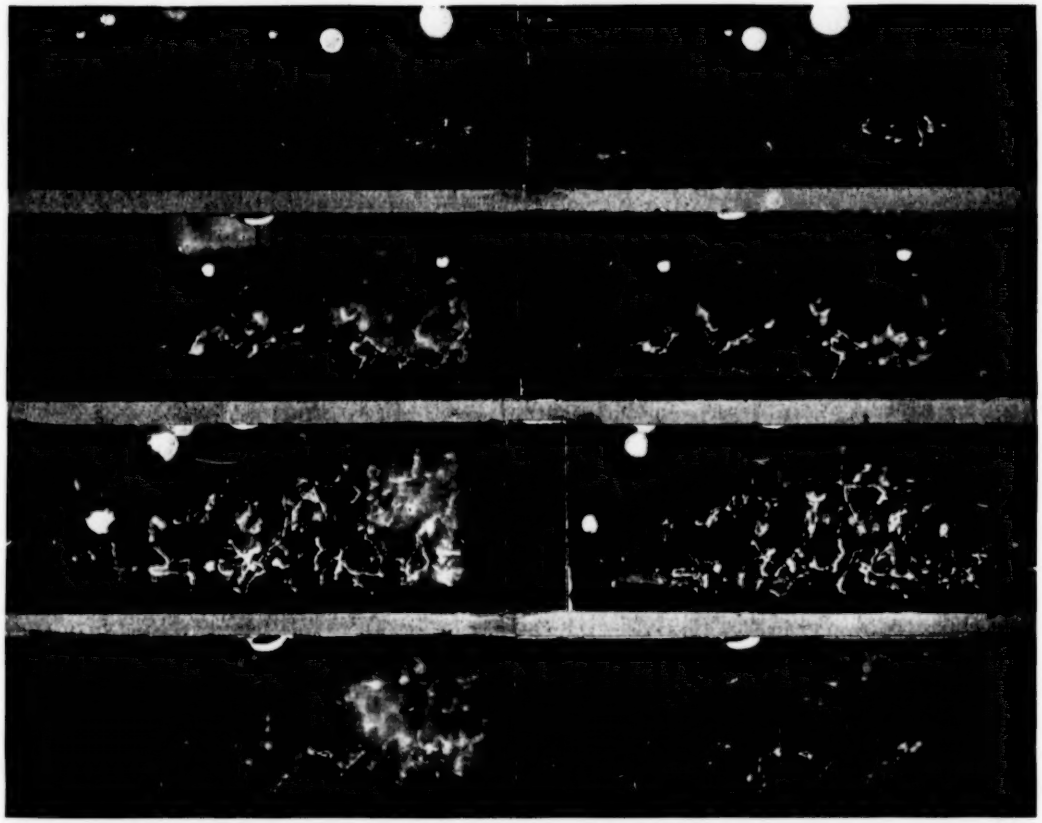


PLATE I

Stereoscopic photographs by Wilson's cloud method of tracks in air of photo-electrons ejected by polarized x-rays. These exhibit: (1) *Lateral asymmetry* in that most of the tracks have initial directions nearly parallel to electric vector of polarized x-rays (see 45° line); (2) *Longitudinal asymmetry* in that most of tracks have forward component with beam. (Use stereoscope.)

necessity of using a bundle instead of a line of x-rays, which renders it impossible to scatter the whole bundle at 90°, have the effect of decreasing the degree of polarization. Allowing for these disturbing factors, A. H. Compton and C. F. Hagenow have shown[4] that hard x-rays, such as were used in these experiments, when scattered at 90° by light elements, are, within the limits of experimental error, completely plane polarized. By comparison with these writers' experiments, it appears that about 85 per cent of the energy of the beam used in the present experiment may be considered as completely polarized, the remaining 15 per cent being unpolarized. If we suppose the 15 per cent unpolarized portion to eject with equal probability in all directions perpendicular to the beam, its effect is shown by the straight line *C* whose ordinate at 360° is 15 per cent of the corresponding ordinate to curve A. The curve B is then obtained by subtracting the ordinates of line *C* from those of curve A.

The curve D is the first derivative of the curve B, and gives the relative number of tracks per degree for all angles between 0° and 360°. The curve D has two well defined lines or humps with maxima at 48° and 228°. These two angles differ by 180° and fix a single line parallel to the electric force of the radiation. Consequently the curve D shows a strong concentration of photo-electrons ejected nearly parallel to the electric force of the polarized beam.

Since these experiments were performed there has appeared an abstract[5] of a paper recently read before the Royal Society by C. T. R. Wilson who discusses some photographs of secondary β-ray tracks which he has obtained by his cloud method. Wilson notes that "partial polarization of the primary beams is indicated by the direction of ejection of a number of the β-particles being in one plane—that containing the direction of the cathode rays in the x-ray tube." Since the plane containing the cathode stream and the beam of x-rays also contains the electric vector of the polarized portion of the radiation, this observation may be taken as confirming the lateral asymmetry brought out by curve D.

Stereoscopic examination of the photographs gives one-sixth of the tracks with an initial component opposite to the beam, one-third approximately at right angles to the beam, and one-half with a component along the beam. In his abstract[5] Wilson remarks that "of the ordinary long range tracks, the majority have a forward component comparable with the lateral component." In this case Wilson's method of taking the photograph with the plate parallel to the beam is better adapted to careful observation than the present method with the plate perpendicular to the beam, although the reverse is true in showing the lateral asymmetry.

[4] A. H. Compton and C. F. Hagenow, Phys. Rev. **18**, 97 (1921)
[5] C. T. R. Wilson, Nature, July 7 (1923)

This effect is probably the same as that first noted by Mackensie[6] and investigated by Bragg,[7] Stuhlmann,[8] Kleeman[9] and Cooksey[10] who found that radiation ejects more photo-electrons from that side of a thin plate from which the radiation emerges than from that side upon which the radiation is incident. Cloud experiments are now in progress in which the ratio of emergent to incident photo-electric current is being measured by counting the forward and backward tracks. In these experiments various gases and various homogeneous x-rays are being used. This method seems likely to give more reliable quantitative results than any method hitherto employed since scattering of the photo-electrons subsequent to ejection is eliminated.

Discussion

The results are in accord with the classical theory. If we adopt the view however that a light quantum consists of a concentrated bundle of energy, such as for example Bragg's "neutron," the present experiment shows that this corpuscle or quantum is not a scalar bundle of energy but has rather definite vector properties. For it explodes, so to speak at right angles to its direction of motion and in a definite plane. Furthermore from polarization phenomena we conclude that such a "vector quantum" once started on its way, maintains its vector properties constant in direction.

It seems likely that the lack of exact coincidence between the direction of the electric force of the radiation and the direction of ejection of the photo-electron may be explained by taking into account the momentum of the electron in its atomic orbit at the instant it is given an impulse in the direction of the electric force of the radiation by the exploding quantum. If we further take into account the forward momentum $h\nu/c$ of the quantum, the longitudinal asymmetry may be explained qualitatively. Calculations based on these ideas together with some experimental evidence in their support will soon be presented.

The writer wishes to make grateful acknowledgment to Professor A. H. Compton for constant advice and assistance, and for placing the facilities of the Washington University Physical Laboratory at his disposal.

Washington University,
St. Louis, Mo.
August 23, 1923.

[6] Mackensie, Phil. Mag. **14,** 176 (1907)
[7] W. H. Bragg, Phil. Mag. **15,** 663 (1908)
[8] O. Stuhlmann, Phil. Mag. **22,** 854 (1911)
[9] R. D. Kleeman, Nature, May 19 (1910)
[10] C. D. Cooksey, Phil. Mag. **29,** 37 (1912)

SYMMETRY OF INCIDENT AND EMERGENT PHOTO-ELECTRONIC VELOCITIES

By R. J. Piersol

Abstract

Relative maximum velocities of incident and emergent photo-electrons from a thin Pt film.—Previous observers have obtained ratios of emergent to incident energies of 1.25 or more, contrary to the quantum theory, but these ratios are shown to be due to the rounding off of the feet of the current voltage curves as a result of the presence of stray light of shorter wave-length. When monochromatic light was used and precautions were taken to avoid stray light, a photo-electric cell of the ordinary type with a thin Pt film deposited on glass as target, gave straight line curves crossing the line of zero current at the same voltage point. Additional experiments showed that the maximum energy is the same in all directions and equal to 1.76 volt for 1850 A and .20 volt for 2540 A.

RECENTLY the writer[1] has published preliminary results which show a difference of velocity between the incident and emergent photo-electrons less than experimental error. Previous to the accurate determination by photo-electronic velocity by Millikan[2] of Planck's constant, a large amount of inexact data was presented in an attempt to prove that if the quantum theory applied to the photo-electric effect at all, then there must be two values for Planck's constant, one obtained by incident and the other by the emergent photo-electronic velocity.

The asymmetry of incident and emergent photo-electric currents was first discovered by Rubens and Ladenburg[3] in gold leaf. These results were substantiated by Kleeman,[4] Stuhlmann,[5] Robinson[6] and Hughes[7] using thin cathode films of different metals deposited on quartz.

At a later date it was shown by Beatty[8] that Röntgen rays on passing through thin films produce a greater ionization current on the emergent side than the incident side, but that the velocity of the electrons on both sides is the same. On the other hand Robinson,[9] in studying the veloci-

[1] Piersol, Phys. Rev. **20,** 195, 1922
[2] Millikan, Phys. Rev. **7,** 355, 1916
[3] Rubens and Ladenburg, Ber. d. Deutsch. Phys. Ges. **24,** 749, 1907
[4] Kleeman, Proc. Roy. Soc. **84,** 92, 1910
[5] Stuhlmann, Phil. Mag. **20,** 854, 1911
[6] Robinson, Phil. Mag. **23,** 542, 1912
[7] Hughes, Phil. Trans. Roy. Soc. **212,** 205, 1912
[8] Beatty, Phil. Mag. **20,** 320, 1910
[9] Robinson, Phil. Mag. **25,** 115, 1913

ties, obtained results which seemed to show that not only is the emergent photo-electronic current greater but also the emergent photo-electronic velocity. Stuhlmann[10] repeated this work using Robinson's design of apparatus. His results show asymmetric velocities. In explanation Stuhlmann stated that the photo-electric effect depends on occluded gases since it was supposed that the photo-electric effect could be obliterated by removal of occluded gases. Later the writer[11] showed that such an explanation is not valid because a metal does have an intrinsic photo-electric effect which is independent of the effect due to occluded gases.

In using the Einstein equation, Richardson and Compton[12] from photo-electronic measurements found the value of h to be from 15 to 40 per cent less than the accepted value. Hughes attempted to explain this on the supposition that the energy of emission opposed to the direction of light, is less than the energy in the direction of incident light. This attempted explanation caused a large volume of research in the attempt to show that the emergent photo-electronic velocity would satisfy the quantum constant.

Probably the most important innovation by Millikan in his emission velocity determinations is that he did not attempt to locate experimentally that retarding potential which would just prevent the escape of an electron from the illuminated metal to the receiving gauze. It is obvious that errors due to the reflected light, reflected electrons, stray light, or insulation leakage which would not produce one half of a per cent error as compared to the value of the saturation current will produce as much as 100 percent error with a current approaching null value. Instead of direct readings Millikan plotted the values in the distribution of the velocity curve in the neighborhood of the null point and then located that point graphically by extending the curve backward to obtain the abscissa intercept.

If Millikan's extrapolation method is applied to the results of Robinson and of Stuhlmann the ratios come out approximately unity instead of 1.16 and 1.42.

Part I. Ratio of Incident and Emergent Velocities

In order to analyze the conclusions reached by Robinson and Stuhlmann the best method of attack would seem to be an attempt to duplicate their experimental data as to velocities of emission. Thus the apparatus as shown in Fig. 1, is almost an exact duplication of the type of photo-

[10] Stuhlmann, Phys. Rev. **4**, 195, 1914
[11] Piersol, Phys. Rev. **8**, 238, 1916
[12] Richardson and Compton, Phil. Mag. **24**, 575, 1912

electric cell used previously. The outer brass cylinder A is 8×5 cm inside measurements. The inner concentric copper gauze cylinder B, 7×4 cm, is supported by a brass rod C through a ground glass joint D

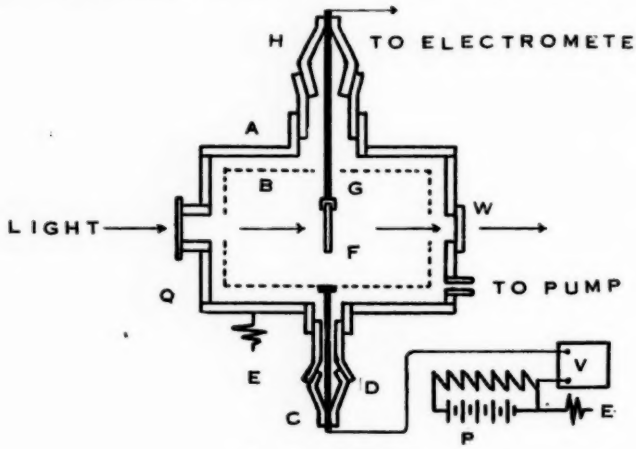


Fig. 1. Apparatus

and is connected to the potentiometer system P with a standard Weston voltmeter V reading to 0.002 volts. The ultra-violet light from a Cooper Hewitt quartz mercury arc, condensed by a quartz lens, passes through the quartz window Q and out the window W. In a position perpendicular to the light ray is placed a quartz plate F, which may be covered with a sputtered film. The plate is attached to a supporting brass rod G with a screw clamp. The rod passes through a ground glass joint H to an electrometer, which has a sensitivity of 1200 divisions per volt at a scale distance of 1 meter. The ground glass joint is used to rotate the filmed quartz plate so that it may be on the side of the quartz incident to the light or in the opposite 180° position in which the electrons will be emitted in the direction of the light. The inside of the cell is covered with camphor soot to reduce the reflected light. Since the two windows are parallel to the quartz plate, the light directly reflected from the film in either the incident or emergent position is sent back through the window Q.

The platinum films are deposited by a platinum cathode, using aluminum as an anode. A Snook-Röntgen machine is used to obtain high voltage direct current. The sputtering is done in a bell jar under a vacuum of 0.001 mm of mercury. Precautions are taken to exclude vapor from stop-cock grease, which may cause spuriously high velocities. A gold leaf mercury vapor trap is introduced between the bell jar and mercury pump. With constant cathode fall and constant current Tyndall

and Hughes[13] have shown that the thickness of the film is a linear function of the time of deposition. Robinson[14] has shown that the maximum ratio of emergent to incident current takes place at a film thickness where 16 per cent of the incident light is absorbed by the film. Since the object is to investigate the maximum asymmetry of velocities, all films used are of this critical thickness, of the order of 10^{-7} cm.

Both Compton[15] and Shaw[16] have shown that an ionization method may be used to determine the contact difference of potential between the platinum film and the receiving copper gauze. A tube containing 0.98 mg of RaCl is placed near the window of the cell and the potential to which the copper gauze must be raised to give no electrometer deflection is measured.

Immediately after sputtering, the films were very erratic as to contact difference of potential, but after several hours they dropped to a very uniform value. None of the films were used until they had aged at least one week.

Fig. 2 shows the first set of results, which were obtained under atmospheric conditions. Distribution of velocity curve agrees with those

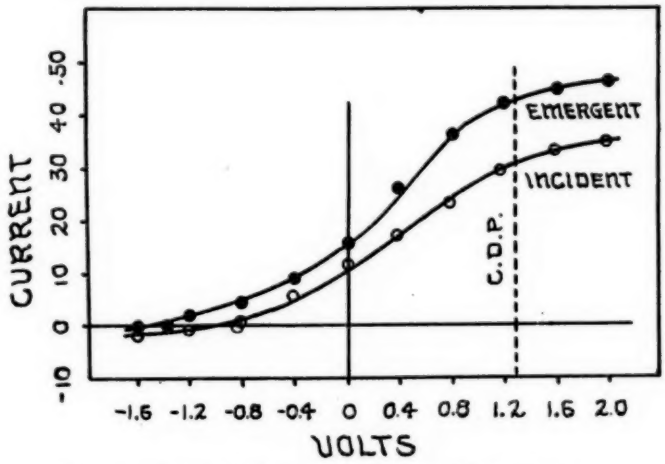


Fig. 2. Photo-electric current at atmospheric pressure.

obtained previously, showing an emergent current almost one-half greater than the incident current. Measuring the stopping potential, the emergent energy is 1.34 volts, the incident energy, 0.81 volts. The contact difference of potential is 1.27 volts. This gives a corrected

[13] Tyndall and Hughes, Phil. Mag. **27**, 415, 1914
[14] Robinson, Phil. Mag. **32**, 421, 1916
[15] Compton, Phil. Mag. **23**, 579, 1912
[16] Shaw, Phil. Mag. **25**, 241, 1913

stopping potential of 2.61 volts for the emergent energy and 2.08 volts for the incident energy or a rat'o of 1.27.

Fig. 3 shows the same film under the lowest pressure which can be obtained by a Gaede rotary mercury pump, which a McLeod Gauge shows to be of the order of 10^{-6} cm. Again the incident current is much less than the emergent current. The contact difference of potential may be obtained graphically by drawing a straight line through the ascending values of the current and another straight line through the constant saturated values of the current. The interceptof these two lines gives the position of the contact difference of the potential which is 1.50 volts.

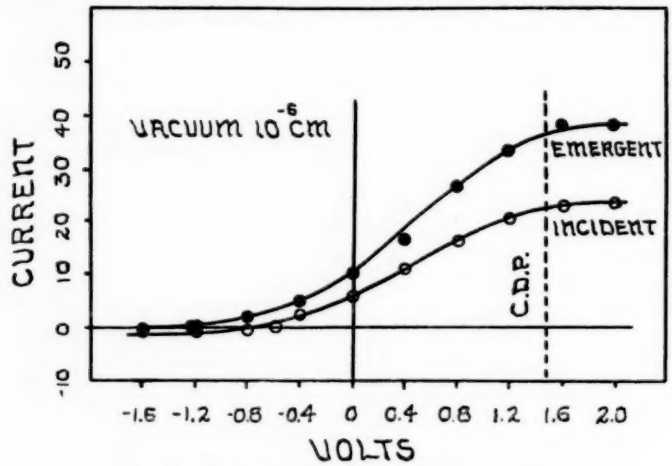


Fig. 3. Same film as Fig. 2 in high vacuum.

This shows that the contact difference of potential has shifted due to the evacuation. Since this is true, the ionization method previously described may not be reliable in the case of a vacuum. The saturation current is very constant. The energy potential for the emergent electrons is 1.23 volts as compared to the incident value of 0.60 volt. Correcting for the contact difference of potential the ratio is 1.30.

Fig. 4 shows the distribution of velocity curve under the critical pressure of 10^{-3} cm which gives about ten times the current obtained either under high vacuum or atmospheric pressure. This condition is valuable for observation as pointed out by Millikan[17] because it increases the precision of observation to within one per cent. It will be noted that the contact difference of potential is again shifted; also that the saturation current has a slight uniform increase with voltage. The emergent potential is 1.42 volts; the incident potential, 0.98 volts. The contact difference of potential is 1.15 volts thereby giving a ratio of 1.21.

[17] Millikan, Phys. Rev. **7**, 374, 1916

Fig. 5 shows the results of using monochromatic light as obtained by a Hilger spectrometer of the reflecting type, using wave-length of 2537 A. The critical pressure of 10^{-3} cm is used. The curves still show "feet" to

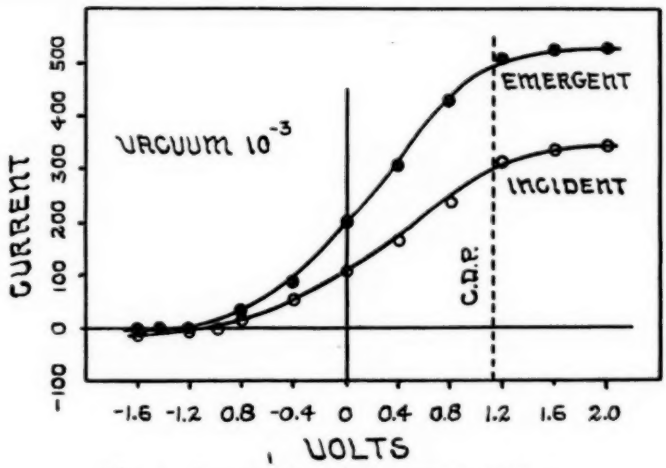


Fig. 4. Curve with critical pressure of 10^{-3} cm.

a marked degree. The characteristic of the curves is that the current increases very rapidly, showing that 2537 A is close to the threshold wave-length of platinum sensibility. The value estimated by Koppius[18]

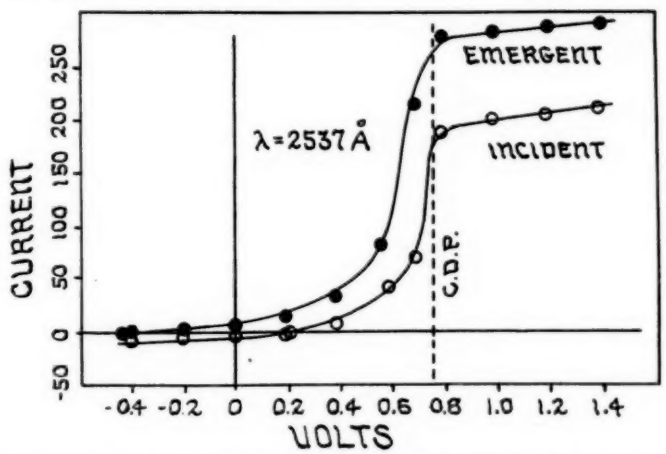


Fig. 5. Curve for monochromatic light 2537 A, pressure being 10^{-3} cm.

is 2570 A. The emergent velocity is −0.43 volts; the incident velocity, 0.21 volts. The contact difference of potential is 0.75 volts, with a resulting ratio of 2.19. Obviously this large ratio is associated with the fact

[18] Koppius, Phys. Rev. **28**, 443, 1921

that the current is asymptotic to the voltage axis. So if the influence of stray light were neglected it would appear that monochromatic light of 2537 A gives a larger divergence in emergent and incident velocities than that obtained by white light.

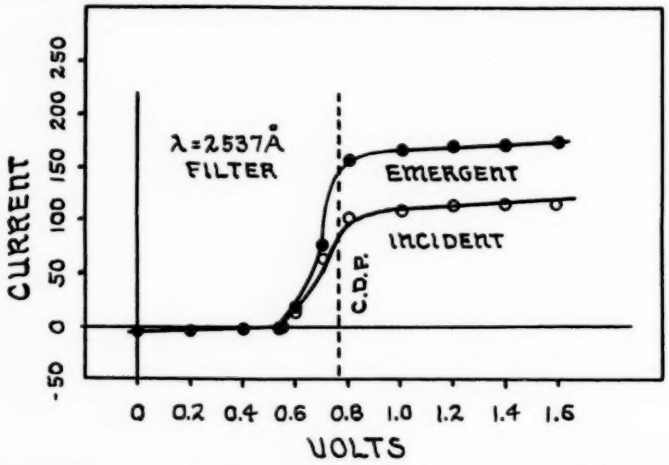


Fig. 6. Curve for monochromatic light 2537 A filtered to eliminate stray light.

In work on absorption media which has not been published, T. M. Dohn has found that a solution of thiophine transmits 2537 A and cuts

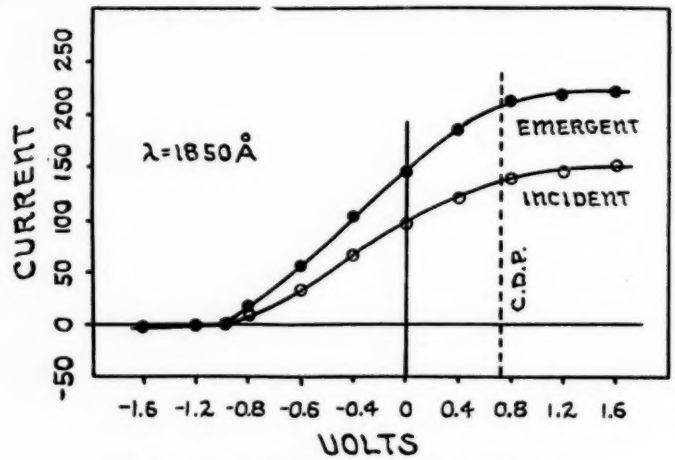


Fig. 7. Curve for monochromatic light, 1850 A.

out the shorter wave-lengths given by a mercury vapor arc. Fig. 6 shows the result of such a filter on the previous curve in Fig. 5. The "feet" are entirely absent. The stopping potential for the emergent velocity is 0.56 volts and for the incident, 0.56 volts, showing direct

experimental symmetry of velocity. The graphical method gives an emergent and incident intercept at 0.57 volts. This seems to prove that the experimental value of the stopping potential approaches the true graphical value only when the feet have been removed by filtering out the shorter wave-lengths.

As a check on this assumption, the shorter wave-length, 1850 A was used. This is the lowest intense line from the mercury arc which is not absorbed by the quartz. Fig. 7 shows the results. The feet are entirely absent as practically all the energy from the shorter wave-length have been cut off by the quartz. In this case the experimental stopping potential corresponds to the graphical intercept, either method giving a ratio of unity.

Although this article is concerned only with the maximum value of stopping potentials, it may be observed that the distribution of velocity is nearer linear than usual. This should be considered a characteristic of the particular photo-electric cell rather than a contradiction to the usual Maxwell distribution of velocity.

PART II. VARIATION OF VELOCITY WITH ANGLE

In order to study the velocity at different angular positions a guard ring type of photo-electric cell is designed as shown in Fig. 8. The outer vertical cylindrical case A is grounded to serve as a magnetic

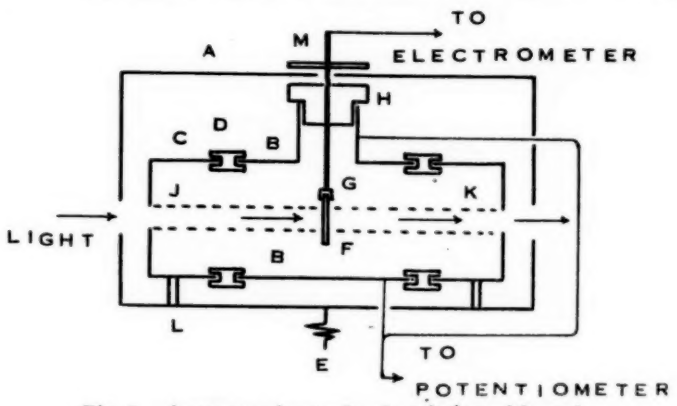


Fig. 8. Apparatus for study of variation with angle.

shield. The inner cylinder is concentric; the electrode B being separated from the grounded guard ring C by sulphur insulators D. The inner cylinder, at its guard ring, is attached to the outer cylinder by conducting supports L. The platinum film quartz plate F is supported by the rod G, passing through a sulphur support H, to the electrometer. Fixed to the rod G is a circular scale M, which reads the

angular position of the film, the sulphur plug rotating in its support. The receiving electrode is connected to the potentiometer P. Monochromatic light, from the vapor quartz mercury lamp passing through the Hilger spectrometer, enters in the window J and the emergent ray passes out through the window K. Since the film is to be rotated through 360 degrees the reflected light will strike the circumference throughout its entire angular position. To overcome this the cylinder is made in two halves with a peripheral slit window being continuous from K to J with the exception of rods to support the upper cylinder so placed that the light reflected at 10° will not strike them. The inner surfaces are coated with camphor soot to absorb all stray light.

The velocity for each angle is obtained by plotting the current-voltage intercept. Since the tests are made in air the contact difference of potential is ascertained by the ionization method. Within experimental error the velocities are constant for all incident and emergent angular positions and equal to 0.20 volts.

Since the emission is low at 2537 A another curve was run for 1850 A. The magnitude of the voltage is about ten times that of the previous curve, being 1.76 volts.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF PITTSBURGH,
July 3, 1923.

ELECTRON EMISSION FROM METALS AS A FUNCTION OF TEMPERATURE

By O. W. Richardson

Abstract

The equation $i = AT^{\frac{1}{2}}e^{-b_0/T}$ was first presented in 1911, and the fact that A is a universal constant was stated in 1915. The proof involved nothing beyond the application of elementary thermodynamics to the equilibrium of the external electron atmospheres, hence the development owes nothing to Nernst's heat theorem or chemical constants. Contrary to Dushman's implication (Phys. Rev. **21,** 623, 1923), the writer did not consider it probable that the work function is the same for all substances.

IN his interesting and valuable paper in the June number of the Physical Review, Mr. Dushman gives an account of the historical genesis of the thermionic electron emission equation

$$i = AT^{\frac{1}{2}}e^{-b_0/T} \tag{1}$$

which hardly does justice to the early history of the matter. Although this equation is a more exact approximation than the older equation

$$i = A_1T^{\frac{1}{2}}e^{-b/T} \tag{2}$$

which is generally regarded as exact for all practical purposes, it is nevertheless in the mathematical sense an approximate equation. Subject to the implications of this limitation A is a universal constant and b_0 a constant characteristic of the material but independent of T.

I first published this equation in a paper presented to the Chicago Meeting of the American Physical Society in December 1911 of which an abstract is to be found in the January number of the Physical Review for 1912, p. 78. This paper was subsequently printed in the Philosophical Magazine, vol. 23, p. 594 (1912). The relevant part of the paper is mainly on pp. 603–605 and 619. My demonstration was based on the application of a Carnot's cycle to the attenuated electron atmospheres in equilibrium with a piece of metal whose ends were maintained at different temperatures. For other publications dealing with the matter at about this date reference may be made to a paper in the Philosophical Magazine, vol. 24, p. 740 (1912) and to the 1st Edition of my book "The Electron Theory of Matter" pp. 450–455 (1914). I am not sure when I first noticed that the A must be a *universal* constant in the new equation, but I find that I stated this in a paper in the Proceedings of the Royal Society A vol. 91, p. 530, Eq. (5). This paper was received by the

Royal Society on April 22nd, 1915. On p. 42 of the first edition of my book "The Emission of Electricity from Hot Bodies" (London 1916), I gave a proof that A is a universal constant. This involves nothing beyond the application of elementary thermodynamics to the equilibrium of the external electron atmospheres. In the same chapter the whole thermodynamic basis of the formula is made clear.

It is not possible to derive an exact equation for the saturation current at present, because this would require a knowledge of the laws of electron reflexion by metals which we do not yet possess. I have, however, in the publications referred to above, given the exact equations for the equilibrium electron pressures and densities which correspond to the approximate Eq. (1) for the saturation current. These equations likewise include in the same way a universal constant.

As the dates of these publications anticipate by several years any publications on these matters in which reference is made to Nernst's heat theorem or chemical constants, it is clear that the historical development owes nothing to Nernst's heat theorem or chemical constants as regards either (a) Eq. (1) as it stands, (b) the equivalent exact equations for the pressures and densities to which I have referred or (c) the fact that the multiplier in these equations is a universal constant. Even as regards the precise physical constitution of this universal constant it does not seem to owe much. As Mr. Dushman has pointed out, I had already in 1914 made an attempt[1] to calculate the thermionic emission on the basis of the quantum theory. Whilst it might be hard to justify all the assumptions underlying this calculation in the present state of our knowledge of the quantum theory it nevertheless led to the correct value k^2me/h^3 for the dimensional factor entering into the universal constant and was only in error by a dimensionless factor of order unity. It appears from Mr. Dushman's paper that the chemical constant experts are not even yet completely unanimous as to the precise magnitude of this dimensionless factor.

I should like to take this opportunity to correct the following statement at the end of Mr. Dushman's paper (pp. 635-6) viz: "On the basis of some measurements of φ obtained by Lester for the elements molybdenum, carbon, tungsten and tantalum, Richardson has suggested that the work function might be found to be the same for all substances." This statement seems to suggest that I thought such a contingency probable. What I actually wrote was "If it could be established generally that φ is the same for all substances most important consequences would follow, amongst others the absence of contact electromotive force under good

[1] Richardson, Phil. Mag. **28**, 633 (1914)

vacuum conditions." This is a true proposition, to the degree of approximation which disregards quantities of the order of the thermoelectric effects, but by stating it I only intended to point out that the conclusion followed from the premises, not that the truth of the premises was probable. Such a position would have been contrary, as Mr. Dushman points out, to the weight of evidence, and he might also have added to the statements, in my book. I confess to some surprise at being accused of holding views precisely opposite to those I have actually held for over a decade. I have evidently overdone my endeavor to be fair to the other side of the question.

These criticisms are intended in no unfriendly spirit. Obviously I have no quarrel with Mr. Dushman's general position and his attempts to determine by experiment the real values of the emission constants of the elements must meet with general approbation.

KING'S COLLEGE,
LONDON, ENGLAND,
October 29, 1923

ELECTRON EMISSION FROM METALS AS A FUNCTION OF TEMPERATURE; ADDED NOTE

By Saul Dushman

Abstract

Recent very accurate measurements on the emission from filaments suggest that the constant A has the same value for all pure crystalline substances such as pure W, Mo and Ta, but is less for thoriated tungsten, caesiated tungsten and oxide covered platinum. This apparent disagreement with theory is briefly discussed.

THE subject of thermionic emission must always be associated with the name Professor Richardson, and his contribution on the historical development of an equation for electron emission is, therefore, of great interest. (See preceding paper.)

Naturally, it seemed unnecessary for the present writer to enter into a full historical discussion in a paper which was mainly concerned with the development of what appeared to be a significant relation between the evaporation of monatomic substances and of electrons.

It may be mentioned in this connection that as a result of a series of very accurate measurements on the emission from thoriated tungsten filaments the writer has been led to the conclusion that the value of the universal constant A, previously derived, is valid only for pure crystalline substances. Thus, for pure tungsten, molybdenum and tantalum, the value of A is found to be experimentally in agreement with the value of 60.2 amp/cm^2 deg.^{2} On the other hand, for fully activated thoriated tungsten the constant has approximately one tenth the above value and increases as the layer of thorium is removed. Similar results have been observed by Kingdon in the case of caesiated tungsten filaments and there is no doubt that the value of A for platinum covered with oxides is also very much lower than that deduced by the writer. This apparent disagreement with the theory is, however, undoubtedly accounted for in the light of G. N. Lewis' interpretation of the third law of thermodynamics. According to the latter the entropy at $T=0$ is zero only for pure crystalline substances, for which the thermodynamic probability is unity. However, for substances in which the atoms are not regularly arranged in a lattice the probability is greater than unity and the entropy must, therefore, be greater than zero. From these considerations it follows that for the latter class of substances the value of the constant A should be smaller than that given above. The full details of the measurements with thoriated tungsten and a discussion of their significance will be published in the near future.

Research Laboratory, General Electric Company,
Schenectady, New York, November 13, 1923.

THE MOBILITIES OF ELECTRONS IN HELIUM

By Leonard B. Loeb

Abstract

Mobility of electrons in helium was determined for fields up to 50 volts/cm and at pressures of 447, 189, 117 and 49 mm of Hg, using, as in previous work, the alternating current method with high frequency oscillations of sine wave form. The He was purified by prolonged circulation over charcoal in liquid air, maintained by a mercury vapor pump. Spectroscopic observation indicated the impurity to be about 1/10,000, chiefly H. The results for the mobility constant, reduced to 760 mm pressure and computed in the ordinary way are well represented by the equation $K'=[11.0\ (10)^8/(2.45+E_0\ 760/p)]^{\frac{1}{2}}$, where $E_0=V_0/d$ is the electric field as read on a static voltmeter. It has been pointed out that since K is a function of E_0, a correction is necessary because of the use of a varying field in the measurements. It is shown that this correction may be made by multiplying the constants in the equation for K' by factors which depend upon the form of the equation. The corrected equation is $K=[7.57\ (10)^8/\ (1.56+E_0\ 760/p)]^{\frac{1}{2}}$. For $E_0=0$, this gives $K_0=22{,}000$, a value close to that given by Townsend's equation. Hence the mean free path seems to have the value given by the Kinetic Theory. Compton's theory agrees well for E_0 above 20 volts/cm, but gives too low values for lower fields.

Mobility of electrons in hydrogen and nitrogen.—*The corrected empirical equations* from previous observations are $K=[3.637(10)^5/(11.9+E_0\cdot 760/p)]$ for N_2 and $K=\left\{4.32(10)^5/[55.2+E_0\ (760/p)^{\frac{1}{2}}]\right\}$ for H_2.

Introduction

THE results of measurements on the mobilities of electrons in N_2[1] and H_2[2] at atmospheric pressures using the Rutherford alternating current method with sine wave form oscillations, have been given in previous papers. From the results obtained it became of interest to determine the mobilities of electrons in He in which K. T. Compton[3] has shown that the fraction of the energy lost by an electron at each impact with a He atom lies close to the theoretical value $f=2m/M=.000268$, (m being the mass of the electron and M the mass of the He atom). The determination of the electron mobilities in helium made in the same manner as those for the gases cited above form the contents of the present paper.

Experimental Procedure

The greatest difficulty involved in the measurements was the attainment of pure He at from 50 to 400 mm pressure in the type of vessel used

[1] Loeb, L. B., Phys. Rev. **19**, 24-37, Jan. 1922
[2] Loeb, L. B., Phys. Rev. **20**, 398-404, Nov. 1922
[3] Benade and Compton, Phys. Rev. **9**, 187, 1918

for an ionization chamber. The conditions imposed on the ionization chambers by the nature of the measurements were as follows: (1) the walls of the chamber had to be conducting, (2) the insulation of the wire leading out of the chamber to the electrometer had to be of amber, (3) the plate source of photo-electrons had to be quite active, and (4) the two plates had to be parallel. The experimental arrangements fulfilling these conditions precluded "baking out" the apparatus so that the contamination of the He by the occluded gases from the walls became a serious disturbing factor.

The new ionization chamber. The ionization chamber used in the later measurements consisted of a small glass box 14.5×9 cm square and 6 cm deep, which was ground to a heavy glass plate. The inside of the box was heavily silvered by Brashear's solution. A quartz window 3.5 cm in diameter ground onto one of the small ends served to admit the

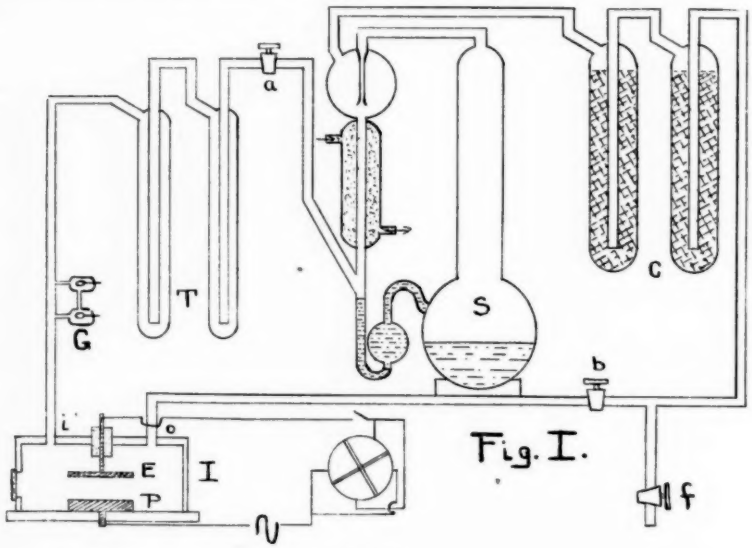


Fig. 1

light from the quartz arc. The photo-electric plate *P* which consisted of a highly polished cylindrical brass block 7 cm in diameter, was screwed onto a brass rod ground through the thick plate glass base. A brass plug ground through the side enabled contact to be made with the silver coating on the inside, by means of a light copper brush. The tubes *i* and *o* were ground into the top of the chamber *I*. The upper plate *E* was fastened to a brass rod ground into an amber plug, which in turn was ground into the thick glass top of the chamber. The plates were adjusted to be as nearly parallel as possible, with a plate distance of 1.641 cm.

After adjusting the plates the apparatus was sealed so as to be gas tight, by gradually heating it up in an oven to about 110°C and running "Express" sealing wax around the outside of the ground joints. To insure complete sealing all the joints were gone over with a small jet-like flame. With this procedure gas-tightness was insured without direct exposure of the space inside to any large quantity of wax. In no place did the wax flow into the chamber, so that the vapors from the wax traversed long leakage paths through the ground joints before diffusing into the apparatus.

Purification of the He. Three separate attempts were made to obtain pure He. In all cases the gas initially came from a large tank obtained from the Navy which contained some four to six per cent of impurities. The principle used in the purification was the repeated circulation of the gas over charcoal cooled in liquid air.

The first attempt which was a failure as far as the results were concerned, consisted in the repeated circulation of the gas stored in a large glass reservoir, over three bulbs of some of Professor H. B. Lemon's specially prepared charcoal cooled in liquid air. This was accomplished by the use of a Toepler pump filled with pure mercury. After the purification in this fashion the gas was run through the charcoal bulbs into the ionization chamber which had been evacuated for several hours by a mercury vapor pump.

In the second purification system the gas was continuously circulated from the ionization chamber to the reservoir and back again through the charcoal bulbs. The results at 447 mm pressure, to be described later, were obtained on gas that had made over 30 complete passages over the charcoal before the measurement was begun.

In the third attempt at purification the large ionization chamber having a volume of seven liters was replaced by a smaller chamber with a volume of but 800 cc. The preliminary purification was made as before using the Toepler pump. Final purification was accomplished in a separate system connected to the ionization chamber, by a continuous flow method using a mercury vapor pump suggested by Dr. F. M. Kannenstine. This purification system is shown diagrammatically in Fig. 1. The ionization chamber I has two plates P and E. The tube i served as the inlet for the gas which came from the Stimson[4] pump S, after passing through a stopcock a, through two mercury traps cooled in liquid air T, and through a Geissler tube of the H type G. The outlet tube o of the ionization chamber I carried the gas through the stopcock b, and through the charcoal bulbs C into the intake of the pump s. It

[4] Stimson, H. F., J. Wash. Acad. Sci. **7**, 477-482, 1917

was found that the pump could be made to circulate well below a pressure of 300 mm, if the tube coming from the boiler and leading to the jet was electrically heated. Owing to the irregular ebullition of the mercury the flow was not smooth; the pressure in the ionization chamber thus varied abruptly over a range of 10 mm so that the flowing gas could not be utilized in measurements.

The actual process of purification therefore consisted in running the partially purified He from the Toepler pump into the circulating system described above, which had been thoroughly evacuated and left standing connected to the charcoal bulbs cooled in liquid air for the preceding twenty-four hours. The pump S was then set into operation and the gas circulated for at least two days before measurements were attempted. When the pump was not in operation (i.e., during the night, or while readings were being made), the stopcocks a and b were closed to keep the gases liberated in C due to the slow evaporation of the liquid air, from getting back into I. The gas was circulated for three hours before measurement. Then the pump was shut off and the readings followed in general within half an hour. In no case were measurements made longer than eight hours after the circulation ceased.

Purity of the helium attained. The purity of the gas was tested spectroscopically using a large Schmidt and Haensch direct-vision spectroscope and small induction coil operating on 6 volts. In the first attempt at purification the spectrum was examined at 20 mm pressure. It showed besides the He lines strong mercury lines from the mercury seal in the old ion chamber. It also showed hydrogen lines, the Hα line being in general as strong if not stronger than the red He line of wave-length 6678 A. No bands were observed unless the Geissler tube was run for a minute when the discharge heated the tube to about 50° C. The gases causing these bands doubtless came from the walls and electrodes of the Geissler tube, and gave two prominent bands in the green associated with the spectrum of CO.

The values of the mobilities obtained with this gas indicated contamination from the chamber in a marked degree. If the gas stood in the chamber more than 24 hours the value of the mobility observed increased as much as 10 per cent. Furthermore, measurements made on the freshly purified gas taken a week apart showed a progressive decrease towards lower values, which did not appear to approach a definite limit. It was for this reason that after some seventy measurements made with this arrangement the results were discarded, and the method of purification modified.

In the second attempt the spectroscopic tests at 20 mm pressure showed no marked improvement, and in fact the two very faint CO bands appeared to be slightly more prominent than before. The mobilities were now, however, constant over several days and showed no tendency to change on further purification. The values lay well below those obtained before. They are recorded for a pressure of 447 mm. in curve 1, Fig. 2. The lack of spectroscopic improvement led to the third more radical attempt at purification of the gas.

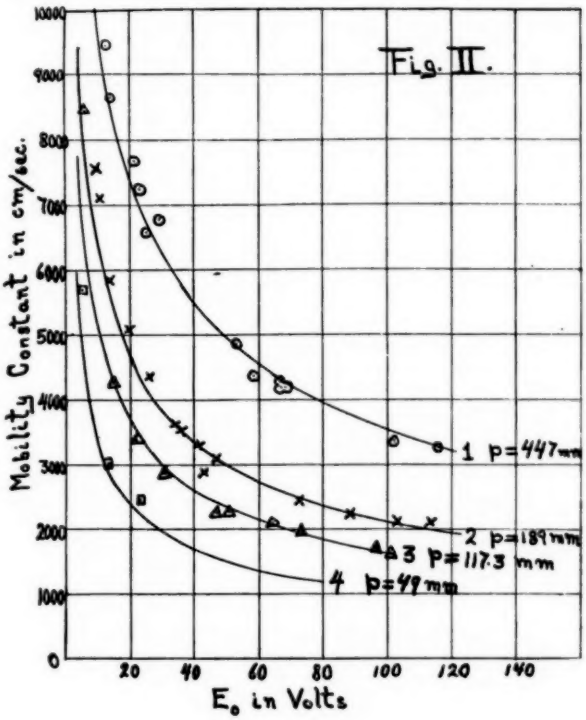


Fig. 2

In this case the gas was examined spectroscopically at 50 mm, 20 mm, and at a pressure less than a mm. In the latter case the gas came directly from the charcoal bulbs immersed in liquid air and nothing but the He lines appeared. The gas studied at 50 mm, while it was circulating, or just after the pump was shut down, showed the green Hg line 5461 A very faintly. The Hα line was about one-fourth as intense as the red He line, 6678 A (sometimes even fainter), and the CO bands were so faint as to be barely perceptible to an eye well accustomed to the dark, even on continued passage of the discharge through the Geissler tube. (From the appearance of the spectrum one might estimate the relative

quantity of impurities in the gas to lie below one part in 10000). The measurements made with this system on several different samples of He showed no change in mobility, and were thoroughly consistent among themselves. The results obtained at 189, 117 and 49 mm are shown in curves 2, 3 and 4 of Fig. 2.

Except for the use of the new ionization chamber in some of the measurements, the apparatus and procedure were precisely that used in the case of the electrons in H_2.[2] In the present experiments, however, the use of retarding fields, suggested in the previous paper, was *occasionally* resorted to. The method for computing the mobility when a retarding field is employed is described in a forthcoming paper on the mobilities of electrons in air.[5] The values found for He with and without such fields were so closely the same that it was impossible to distinguish between them.

Values of the Mobility

The points plotted in the curves marked 2, 3, 4 of Fig. 2 are the mobility constants computed from the relation $K=(\pi\, d^2/\sqrt{2}\, E_0 T)(760/p)$ (i. e. the values of the mobility reduced to atmospheric pressure), plotted against the value of the critical voltage E_0. These were obtained with the small ionization chamber using a plate distance of 1.641 cm. The points on the curve 1 were taken at 447 mm with the plate distance of 1.95 cm in the large ion chamber, during the second attempt at purification.

The smooth curves drawn through the points represent the equation

$$K=\sqrt{\frac{11.0\times 10^8}{2.45+(E_0/d)(760/p)}}$$

with the appropriate values of E_0, d and p inserted. As is seen the curves on the whole fit the points very well. The agreement obtained shows that the mobilities in He may be deduced for any pressure and plate distance by means of this relation. Thus He in contrast to H_2[2] shows the strict proportionality between the theoretical constants a and b (of the previous papers[1,2]) and the quantity $760/p$, which is to be expected from theoretical considerations. It is also to be noted that while N_2 and H_2 do not show the expected theoretical form for the equation for the mobility constant K, (viz. $K=\sqrt{a/(b+E_0)}$) the gas He does. A similar agreement in form between the theory cited and experiment has recently been observed by Wahlin[6] in the case of CO.

[5] Loeb, L. B., Proc. Nat. Acad. Sci., Oct. 1923
[6] Wahlin, H. B., Phys. Rev. **21**, 517-524, 1923

Correction of Mobilities Determined with a Sine Wave

In the preceding papers and up to this point in the present paper, the value of K taken from the experimental points has been assumed to be the true value of the mobility.

In December 1922, H. A. Wilson pointed out that owing to the use of a sine wave alternating potential with the variable electron mobilities, a correction should be made in the value of K deduced from experiment to give the true value. The writer had considered the possibility of such a correction in 1921, but the method of analysis used failed to show the necessity of it. A more careful consideration following Professor Wilson's suggestion showed such a correction to be essential. Wahlin[6] in his recent paper on electron mobilities in CO independently arrived at the same conclusion.

To see how such a correction must be applied one may consider the simple case where the mobility constant K is given by an equation whose form is $K=b-aV$, where V is the voltage and a and b are constants. This is an equation which holds for electrons in air at 66.5 mm pressure.[5] In the discussion it is simpler to leave out the factor reducing the mobility computed at a pressure p, to the mobility constant K, at 760 mm. In what follows one may consider that this reduction has already been made. For a sine wave alternating potential the quantity V takes on the value $E_0 \sin\omega t$ when the electrons can just cross the distance d in one half cycle $T/2$. The condition for electrons crossing the plates may be expressed by the relation:

$$d=\int_0^{\frac{1}{2}T}(E_0/d)K\sin\omega t\,dt$$

where $\omega=2\pi/T$

or

$$\frac{d^2}{E_0}=\int_0^{\frac{1}{2}T}(b-aE_0\sin\omega t)\sin\omega t\,dt$$

whence

$$\omega d^2/E_0=2\pi d^2/TE_0=2b-\tfrac{1}{2}\pi aE_0.$$

Now the mobility observed, K', is computed from the relation $K'=\pi d^2/TE_0$. Hence $K'=b-\frac{1}{4}\pi aE_0$. But from the observed data we deduce a relation $K'=b-a'E_0$ whereas actually $K=b-aE_0$. Thus to get the true value of K from the observed value we must divide the a' observed by $\frac{1}{4}\pi$, i.e. we must write $a=(4/\pi)a'$.

In the case of the gases H_2 and N_2 where we found K to be expressed by an equation of the form $K=a/(b+V)$ the correction is not as simple as the one above. Putting this value of K into the condition for electrons going a distance d, we get:

$$d=\int_0^{\frac{1}{2}T}\frac{aE_0}{b+V}\sin\omega t\,dt=\int_0^{\frac{1}{2}T}\frac{aE_0\sin\omega t\,dt}{b+E_0\sin\omega t}$$

or $$d^2 = a\int_0^{\frac{1}{2}T} \frac{\sin \omega t\, dt}{b/E_0 + \sin \omega t}$$

Call $\omega t = \theta$, $B = b/E_0$, and the equation becomes

$$d^2 = \frac{a}{\omega}\int_0^{\pi} \left(1 - \frac{B}{B+\sin\theta}\right) d\theta$$

This integral has two values depending on the value of B

For $B>1$ $$\int_0^{\pi} \frac{d\theta}{B+\sin\theta} = \frac{2B}{\sqrt{(B^2-1)}}\left(\tfrac{1}{2}\pi - \tan^{-1}\frac{1}{\sqrt{(B^2-1)}}\right)$$

For $B<1$ $$\int_0^{\pi} \frac{d\theta}{B+\sin\theta} = \frac{B}{\sqrt{(1-B^2)}} \log \frac{1+\sqrt{(1-B^2)}}{1-\sqrt{(1-B^2)}}$$

From these functions it is impossible to separate out the variable B in a form so as to permit its being applied to the correction of the observed constants a' and b'. By plotting the integrals as a function of B, it is found that the curve obtained may be very closely approximated by the purely arbitrary equation of the form $1.785/(0.568+B)$. The accuracy of the fit is as close as the errors involved in the mobility measurements permit one to work. Thus one can write that

$$\int_0^{\pi} \frac{\sin\theta\, d\theta}{B+\sin\theta} = \frac{1.785}{0.568+B}$$

and hence one may set

$$d^2 = \frac{a}{\omega}\left(\frac{1.785}{0.568+B}\right) = \frac{2E_0}{\omega}\left(\frac{.893\, a}{b+0.569E_0}\right)$$

or $$\frac{\pi d^2}{TE_0} = K' = \frac{1.57a}{1.766+E_0} = \frac{a'}{b'+E_0}.$$

But the true K is given by $K = a/(b+E_0)$, while the observed K is given by the equation above, hence one can write that the observed a' divided by 1.57 gives the true a, and the observed b' divided by 1.76 gives the true b. So that the equations obtained for N_2 and H_2 must have their constants corrected by multiplying the a' observed by .637, and the b' observed by .568. Hence the electron mobilities in H_2 and N_2 now become:

$$K = \frac{363{,}700}{11.9+(V_0/d)(760/p)} \text{ (for } N_2\text{) and } K = \frac{4.32+10^5}{55.2+(V_0/d)(760/p)^{\frac{1}{2}}} \text{ (for } H_2\text{).}$$

The values of K_0 when $V_0 = 0$ become:

$K_0 = 30{,}500$ cm/sec. (for N_2) and $K_0 = 7820$ cm/sec. (for H_2).

Finally in the case of He the correction is derived from the condition that the electrons cross when

$$d=\int_0^{\frac{1}{2}T}\sqrt{\frac{a}{b+E_0\sin\omega t}}\,dt\,\frac{E_0\sin\omega t\,dt}{d}$$

for in He, K varies as $\sqrt{a/(b+V)}$. Putting $B=b/E_0$ and $\omega t=\theta$ we get the relation:

$$d^2=\frac{\sqrt{aE_0}}{\omega}\int_0^{\pi}\frac{\sin\theta\,d\theta}{\sqrt{(B+\sin\theta)}}$$

The integral is an elliptic integral, which as Wahlin[5] shows on reduction takes the form:

$$\int_0^{\pi}\frac{\sin\theta\,d\theta}{\sqrt{(B+\sin\theta)}}=2\sqrt{2}\{2E(k,\Phi)-F(k,\Phi)\}\qquad(2)$$

where

$$\Phi=\sin^{-1}(1+B)^{-\frac{1}{2}}\text{ and }k=\sqrt{\tfrac{1}{2}(1+B)},$$

for values of B less than unity only. For values of B above unity the relation between B and the value of this integral may be computed graphically. The curve represented by plotting the integral in Eq. (2) against B can be very closely approximated to by the following empirical expression:

$$\int_0^{\pi}\frac{\sin\theta\,d\theta}{\sqrt{(B+\sin\theta)}}\cdot=\cdot\sqrt{\frac{3.69}{0.638+B}}$$

and hence

$$d^2=\frac{\sqrt{a\,E_0}}{\omega}\sqrt{\frac{3.69}{.638+B}}$$

whence

$$\frac{\omega d^2}{2E_0}=\frac{\pi d^2}{E_0T}=K'=\sqrt{\frac{1.445a}{1.566\,b+E_0}}$$

Thus the a' and b' observed experimentally are to be multiplied by .688 and .638 respectively to give the true values of the constants of the mobility equation for electrons in He. The mobility of the electrons in He are accordingly best represented by the equation

$$K=\sqrt{\frac{7.57\times10^8}{1.565+(V_0/d)\left(\frac{760}{p}\right)}}.$$

Discussion of the Results and Interpretation of Equation

From this value of K, the value of K_0, the mobility for vanishingly small fields, comes out to be 22,000 cm/sec. Substituting the value of L the mean free path taken from the kinetic theory of gases, into Townsend's equation for electronic mobility viz. $K=.815eL/mc$, where c is the velocity of thermal agitation, one obtains the value of K_0 as 20,300 cm/sec. On this basis it may be concluded that the electron in He has about the mean free path to be expected from the kinetic theory.

Since the form of the mobility equation for He observed experimentally coincides with the theoretical equation deduced by the writer it is of interest to compare the absolute magnitudes of the observed and computed mobilities. According to the writer[1] the mobility is given by

$$K = .815 \frac{e}{m} \frac{L}{c\sqrt{1+.575\, EeL/mdc^2\sqrt{f}}}$$

Putting $d=1$, $p=760$, $f=.000268$, and L proportional to $4\sqrt{2}\times$ that for atoms in He, the equation reduces to

$$K = \sqrt{54.6\times10^8/(13.2+E)}$$

The curve given by this equation is shown in curve 2, Fig. 3. Curve 1 is the experimental curve, taken from the results cited above. It is obvious that there is no agreement between the two curves except near the point

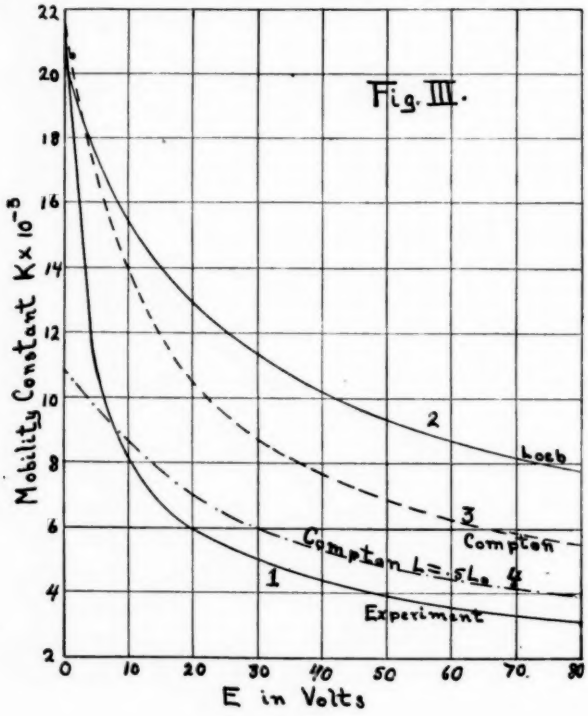


Fig. 3

$E=0$, and except in the fact that their general form is similar. The discrepancy results from the fact that the value of the computed term $.575eL/mc^2\sqrt{f}$ is .0756 which is much smaller than the corresponding factor .64 in the experimental equation. The only uncertain quantities in the theoretical factor are the $\sqrt{f}$ and the L. If the $\sqrt{f}$ were 0.119 times its theoretical value the equations would agree fairly well. This is,

however, impossible as the lower limit that f can have is .000268. It is possible that by lowering L the curves can be made to coincide at higher fields. In doing this the discrepancy is merely shifted to the lower values of E, for the L is in the numerator of the equation for K as well as in the factor discussed above.

In a recent paper K. T. Compton[7] has deduced a slightly different equation for K, in which the averaging of the energy of thermal agitation of the electron and its energy acquired in the field is probably carried out in a more legitimate fashion than in the writer's case. Compton's equation may be written as follows

$$K=\frac{254{,}000\ l_0}{[1+\{1+1.355\times10^6\ Ml_0^2\ (E/p)^2\}^{\frac{1}{2}}]^{\frac{1}{2}}},$$

where l_0 is the electronic free path at 1 mm, p is the pressure in mm, M is the atomic weight of the gas relative to hydrogen as unity, and E is the electric intensity. Putting M equal to 4, l_0 equal to 0.1205 cm, p equal to 760 mm and d equal to unity, this equation becomes comparable to those above, in the form

$$K=\sqrt{\frac{9.37\times10^8}{1+(1+0.1365E^2)^{\frac{1}{2}}}}$$

On plotting this equation the curve 3 of Fig. 3 is obtained. It is seen that while it lies nearer to the observed curve 1 than the writer's equation, the agreement is far from good. If the value of l_0 be taken as 0.32 the kinetic theory value used before, the two curves will lie fairly close together above 20 volts, but differ radically below this. Curve 4 is obtained from Compton's equation putting $l_0=0.0602$ cm.*

The conclusion to be drawn from these results is that while the theory is able to account for the variation of the electronic mobility in its broad outlines, it is in its present form unable to predict the numerical values

[7] Compton, K. T., Phys. Rev. **22**, 333, 1923

* Since this paper was written there has appeared an article on electron mobilities in He by J. S. Townsend (Phil. Mag. **46**, 274, Oct. 1923), in which are given the results of the measurement of electron mobilities in He at pressures up to 240 mm, using his well known lateral diffusion method (Phil. Mag. Dec. 1921). The values obtained by him lie well along the curve 4 (Fig. 3) obtained by putting $l_0=0.0602$ cm in Compton's equation. Since it is probable that the method of Townsend is more capable of giving correct values for K than the writer's owing to the difficulties mentioned in the paper, one may consider this a partial verification of Compton's equation if l_0 have the value assumed. It is desired to point out that the less certain method used by the writer was the first method to give any estimate of the value of the electron mobilities above 100 mm pressure, and is today the only one capable of going above this pressure in most gases. The results given by it are certainly correct in order of magnitude and indicate the variation of these mobilities with field strength and pressure with fair accuracy at the higher field strengths.

obtained for the mobility. It is quite possible that the quantity *assumed to be the mobility in the measurements, is not the true electron mobility*, and thus should not be compared too closely with the theory. The reason for this is that the mobility computed from the measurements is the average of a mobility varying with the field, measured in a variable field. Thus this mobility is an average in which owing to the complexity of the phenomena the averaging may not have been properly performed. There is to be included among the factors making the interpretation of the results uncertain, the added one suggested by Compton[7] that the electrons may travel a finite distance in the electric field before they achieve their terminal speeds. At the lowest pressure used in the present experiments the distance traversed before nine tenths of the terminal speed is acquired is at a maximum seven per cent of the distance between the plates. The error introduced by this effect then is not serious, but should be considered. Thus allowance must be made for the uncertainties in the true values of the mobilities when an attempt is made to compare them with the theory.

National Research Fellowship,
Ryerson Physical Laboratory,
August 7, 1923.

THE MOTION OF ELECTRONS IN NITROGEN

By H. B. Wahlin

Abstract

Mobilities of electrons in nitrogen have been determined in fields ranging from 1 volt/cm to 55 volts/cm at atmospheric pressure. For low fields the mobility has a value of 18,000 cm/sec. but decreases rapidly for higher fields. The results agree for low fields with Compton's mobility equation, viz., $u=a/[1+(1+Bx^2)^{\frac{1}{2}}]^{\frac{1}{2}}$, but in order to obtain full agreement it must be assumed that the mean free path of the electron decreases as the velocity of the electron increases and also that the impacts of electrons with nitrogen molecules are not completely elastic. The *mean free path* of the electrons in nitrogen at a pressure of 760 mm and in thermal equilibrium with the molecules has been determined from the limiting mobility as the field in which the mobility is determined approaches zero. The value thus found is .000131 cm. This is about 2.7 times the kinetic theory value.

SINCE the discovery that when electrons are liberated in a gas they do not become attached immediately to the molecules but in most gases remain free for a considerable time and in some cases never unite to form negative ions, considerable interest has been aroused in the study of motions of electrons through gases under the action of an electric field. The factors involved in such a study are more numerous and somewhat more complicated than those encountered in the case where the negative carriers consist of charged atoms or molecules, for in this case or in any case where the carriers have approximately the same mass as the molecules of the medium through which they are moving, there will be equipartition of energy between the ions and the molecules of the gas, except for the comparatively small amount of energy gained in the field between impacts; therefore, since the free time between impacts is nearly a constant and independent of the voltage, the velocity of the ions in the direction of the field will be proportional to the field intensity. When the carriers consist of electrons only, this ceases to be the case, for due to the great difference in mass between the electrons and molecules of the gas, the electrons will not for the lower speeds lose all the energy gained in the field on impact with the molecules, hence the energy will continue to increase until it reaches a terminal value. This terminal energy will be greater, the greater the field intensity. Consequently, the free time between impacts decreases with the increased field and the velocity in the direction of the field will no longer be proportional to the field strength.

Electronic motion through a gas may be studied most readily by a determination of the mobility of electrons for different values of the field intensity. Townsend and Bailey[1] using a new method in which the velocity in the direction of the field is measured by means of the magnetic deflection of the electron stream, have succeeded in determining not only this velocity but also the factor by which the energy of the electrons exceeds that of the molecules. This method is not very suitable, however, for determining the mobility in low fields or at high gas pressures, that is, for small values of the field strength divided by the pressure. Loeb[2] has applied the Rutherford alternating potential method to the determination of electronic mobilities in a number of gases and has made measurements in nitrogen at pressures ranging from 75 mm to 606 mm, and in fields ranging from 3.8 volts/cm to 52 volts/cm.

Results are herewith presented extending these values from fields of 1 volt/cm to 55 volts/cm, at a pressure of 760 mm and with frequencies ranging from 5300 cycles/sec. to 88,000 cycles/sec.

The method of procedure was essentially the same as that described in an earlier paper.[3] The Rutherford alternating potential method was used and the electrons were obtained photo-electrically from one of the two parallel plates between which the mobility was measured.

As a source of alternating potential, a vacuum tube oscillator of the Hartley type was used and the voltage was determined by means of an electrostatic voltmeter having a range of 30 to 120 volts. A quadrant electrometer having a phosphor-bronze suspension was used to measure voltages below thirty. This was calibrated on direct voltage and had a sensibility such that it gave 250 scale divisions with an applied potential of 30 volts, for determining the mobility in fields down to 8 volts/cm, and a sensibility such as to give a deflection of 170 scale divisions with 15 volts for lower voltages. Since with an alternating potential, the electrometer must be used idiostatically, the deflections for the lower voltages were not very large and an error of 10 per cent could easily be introduced in reading the voltage. For this reason, a number of mobility curves were taken with the same frequency and the mobility taken as the average value found from these curves.

Mobilities were determined with plate distances of 1.55 to 2.4 cm.

A standard wave-meter having a range of 3000 to 23,500 meters was used to determine the higher frequencies. For the lower frequencies, a circuit of the type shown in Fig. 1 was adopted. Here L_1 is an induct-

[1] Townsend and Bailey, Phil. Mag. **42,** 873, 1921
[2] Loeb, L. B., Phys. Rev. **19,** 24, Jan. 1922
[3] Wahlin, H. B., Phys. Rev. **21,** 517, 1923

ance which was coupled inductively with the oscillator; L is a calibrated variable inductance and C a variable condenser of known capacity; T is a pair of phones. L and C were adjusted to give a minimum of sound in the phones, and the frequency was computed from the relation

$$n = 1/2\pi\sqrt{LC}$$

The same values for the frequency were obtained within the limits of error of measurement when L was coupled inductively to the oscillator and the phones placed across C, the circuit being adjusted for a maximum of sound in the phones. The circuit diagrammed seemed preferable,

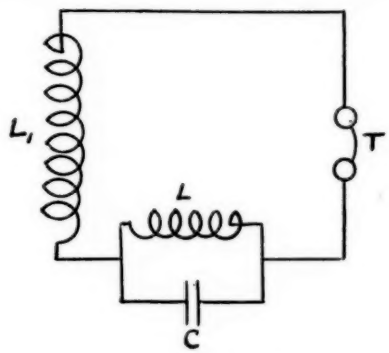


Fig. 1. Circuit used.

however, since it eliminated any possible capacity effect of the phones. From the value of the frequency determined in this way and the value of the voltage for the intercepts of the mobility curves, an approximate value of the mobility may be determined from the equation

$$u = (\pi/\sqrt{2})\ (nd^2/V) \qquad (1)$$

where d is the distance between the plates, V is the effective value of the voltage intercept and n is the frequency.

The nitrogen was obtained from a commercial tank and was purified by passing it over hot copper oxide, hot copper, potassium hydroxide, calcium chloride, phosphorus pentoxide and through two traps cooled with liquid air before it entered the measuring chamber. The chamber was washed four or five times with the purified gas before any readings were taken. This was necessary for otherwise, due to attachment of electrons to impurities, the mobility would decrease in the low fields. Readings were taken immediately after the chamber was filled, so as to eliminate any effect due to impurities diffusing from the walls. However, no change in the mobility was produced by allowing the gas to remain in the chamber for twelve hours and since no set of readings extended over a period as long as this, the effect due to impurities from the

walls was negligible. In order to minimize any effect due to space charge action at the emitting plate, the photo-electric emission was kept as small as was practicable.

The mobility curves were obtained by applying the alternating potential to the emitting plate for a given time (in most cases 20 sec.) and then connecting the electrometer to the collecting plate and measuring the deflection. The voltage was then varied and the process repeated until a sufficient number of readings had been obtained to plot a curve of electrometer deflections against voltage.

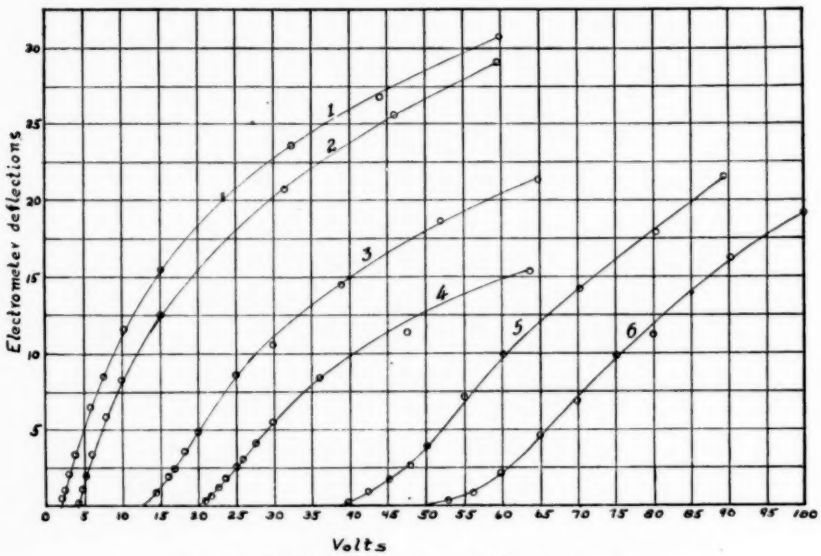


Fig. 2. Mobility curves for various frequencies.

Fig. 2 illustrates the types of curves obtained.

TABLE I

Curves shown in Fig. 2

Curve	Voltage intercept (corrected)	n	d	u
1	1.9	5,300	1.55	17,700
2	3.9	12,850	1.55	17,600
3	13.3	37,500	1.55	15,100
4	20.2	50,500	1.55	13,300
5	40.9	71,100	1.55	9,420
6	52.5	77,000	1.56	7,920

Fortunately the intercepts of the mobility curves are sharper at low voltages than at high ones. If this were not the case, anything but an approximate determination would be impossible, for a small error in determining the intercept in low fields would change the results appreciably.

In Figs. 3 and 4 the values of the mobility computed from Eq. (1) are plotted against the fields in which they were determined.

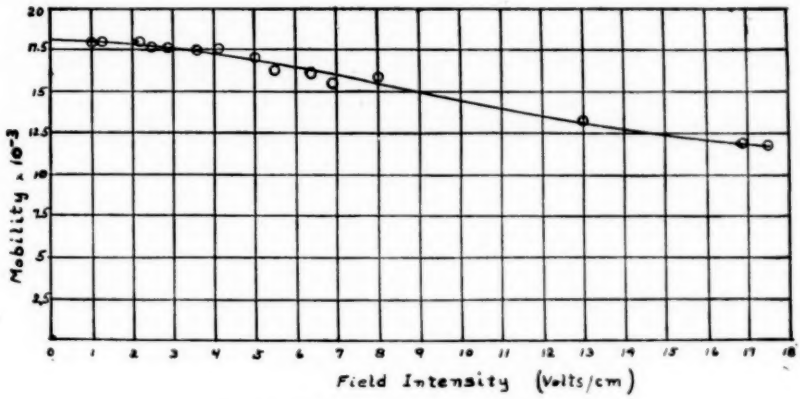


Fig. 3. Mobility for fields to 18 volts/cm.

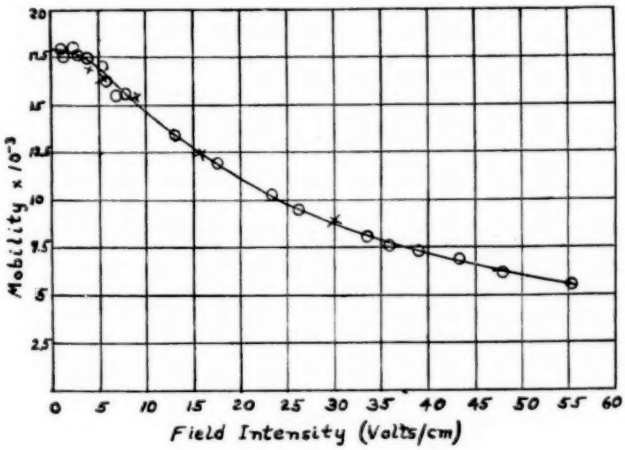


Fig. 4. Mobility for fields to 55 volts/cm.

TABLE II

Mobility as a function of field

V/d	*n*	*u*	*V/d*	*n*	*u*
1.01	5,300	17,900	8.0	37,000	15,900
1.16	6,210	18,100	13.0	50,500	13,300
1.22	6,300	17,700	17.5	60,100	11,850
2.2	11,500	18,000	23.2	68,500	10,300
2.9	15,050	17,600	26.1	71,100	9,420
3.6	18,600	17,400	33.5	77,000	7,950
4.2	21,500	17,500	35.8	79,100	7,520
5.0	25,000	17,000	39.0	81,300	7,250
6.3	29,700	16,200	43.3	84,500	6,800
5.5	16,670	16,200	48.0	85,300	6,110
6.9	30,900	15,500	55.5	87,700	5,500

As will be noticed the mobility in fields below 3 volts is nearly a constant, but decreases quite rapidly as the field is increased above this value. For this reason, there will be an error in the value of the mobility as computed from Eq. (1) for higher fields. This equation is derived from the expression

$$dy/dt = uX \sin 2\pi mt$$

and assumes that u is a constant and independent of the field strength. As may be seen from Figs. 2 and 3 this is not the case, and the equation must be written

$$dy/dt = f(x)X \sin 2\pi nt = f(X \sin 2\pi nt)X \sin 2\pi nt \tag{2}$$

where x is the instantaneous value of the field.

Unless the form of $f(x)$ is known, an exact determination of the mobility using a sine wave alternating potential is, in general, impossible. If the function is known, however, it is possible to compare the frequencies determined experimentally as a function of the voltage intercepts with those computed from theoretical considerations.

K. T. Compton[4] has recently determined the form of $f(x)$ and has found that

$$f(x) = \frac{.815el}{\sqrt{m}[aT + (a^2T^2 + 1.76l^2e^2x^2/f)^{\frac{1}{2}}]^{\frac{1}{2}}} \tag{3}$$

where e and m are the charge and the mass of the electron aT is the mean energy of agitation of the gas, f is the fraction of the electron energy lost at each impact, l is the mean free path and x is the field intensity. This equation can be written in the form

$$u = a[1 + (1 + Bx^2)^{\frac{1}{2}}]^{-\frac{1}{2}} \tag{4}$$

On substituting this in Eq. (2) above we get:

$$\frac{dy}{dt} = \frac{aX \sin 2\pi nt}{[1 + (1 + BX^2 \sin^2 2\pi nt)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

For the case where the electrons just reach the collecting plate during a half cycle of the alternating potential this becomes:

$$d = \int_0^d dy = aX \int_0^{1/2n} \frac{\sin 2\pi nt\, dt}{[1 + (1 + BX^2 \sin^2 2\pi nt)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

Placing $2\pi nt = \theta$ and solving for n this becomes

$$n = \frac{aX}{\pi d} \int_0^{\pi/2} \frac{\sin\theta\, d\theta}{[1 + (1 + BX^2 \sin^2\theta)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

This is an elliptic integral which on reduction gives:

[4] Compton, K. T., Phys. Rev. **22**, 333, Oct. 1923

$$n=\frac{a\sqrt{2r}}{\pi dB}\left[2E(k_1\varphi)-F(k_1\varphi)\right] \tag{5}$$

where $r=\sqrt{BX^2+1}$, $k=\sqrt{(r+1)/2r}$ and $\varphi=\sqrt{(r-1)/(r+1)}$. Since $r \geqq 1$, k is $\leqq 1$ and the solution holds for any value of X.

If a and B can now be determined, n may be plotted as a function of X and the values thus obtained compared with the experimental values.

The constant a can be determined immediately from the limiting mobility as X approaches zero. Eq. (4) becomes for this case:

$$u_0=a/\sqrt{2}$$

Since the experimental value found for u_0 is 18,000 cm/sec./volt/cm, $a=25{,}400$. If it is assumed that the electrons make elastic impacts with the molecules, B may be computed also, for as Compton[4] has shown, Eq. (3) then reduces to

$$u=\frac{.815el}{\sqrt{m}[aT+(a^2T^2+l^2Me^2x^2/1.134m)^{\frac{1}{2}}]^{\frac{1}{2}}}$$

Here M is the molecular weight of the gas.

From the determined value of a, $l=.000131$, and since $M=28$, $B=.658$. These values of a and B are not in agreement with the results of these experiments, as may be seen from Fig. 5, curve 1 where the values of n computed from Eq. (5) are plotted against the effective values of X. The circles represent the experimental values. Curve 2 is the plot of n with $a=25{,}400$ and $B=1/10$ and curve 3 with $a=25{,}400$ and $B=.027$. This last curve agrees quite well with the experimental values for low fields so that Compton's equation may be said to hold in this region. The deviation at higher fields may very well be due to a decrease in the mean free path of the electron with increasing fields. This effect would lower the mobility and consequently the frequency necessary to produce an intercept at a given field intensity. Such a change has been observed by Townsend and Bailey,[1] working with higher fields than those employed here.

If the value of B is taken as .027 it must be assumed in order to obtain agreement with Eq. (3) that f is larger than would be the case if the energy loss on impact were due to a momentum transfer only; in other words, that the electrons make partially elastic impacts with the nitrogen molecules and lose 24.4 (B for elastic impacts divided by .027) times as much energy as they would if the impacts were elastic.

The mean free path of the electrons in thermal equilibrium with nitrogen molecules has been found to be .000131 cm at a pressure of 760 mm. This is about 2.7 times the value determined from kinetic theory con-

siderations which give a value $4\sqrt{2}$ times that of the molecular free path. That there is a tendency for the electronic free path to become abnormally long in nitrogen in low fields may be seen from the results of Townsend and Bailey referred to above.

An examination of the mobility curves in Fig. 2 shows that as the voltage increases, the intercepts of the mobility curves become ill defined and the curves tend to approach the voltage axis asymptotically. This

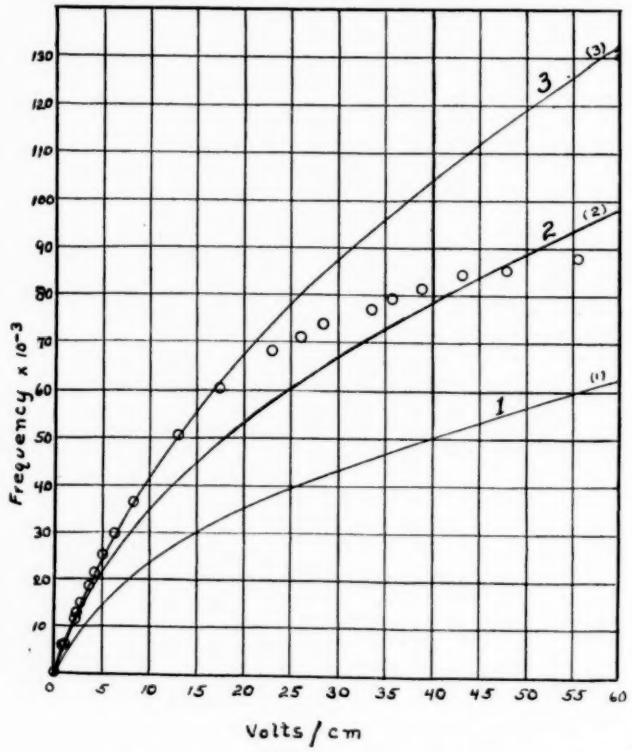


Fig. 5. Comparison of results with theory.

effect was also observed by Loeb[2] who offered as a possible explanation that it might be due to the combination of electrons with impurities in the gas to form ions. These ions would remain in the field and, under a space charge action, would drift across to the collecting plate. Such an explanation will not account for the disappearance of the effect at low frequencies and in low fields. If, however, as seems to be the case, there is a decrease in the mobility due to a decrease in the mean free path with increasing fields and superimposed on this the decrease in the mobility due to the increase in the energy of agitation of the electrons, these effects would combine in such a way as to make the mobility de-

crease more rapidly than it normally would. Consequently, over a small range of field strengths the product of the mobility and the field would be nearly a constant and since this is the velocity of the electrons which in turn is proportional to the space current, the current due to an alternating voltage would tend to become independent of the voltage. This would cause the mobility curves to approach the voltage axis asymptotically. It is significant that this "tailing off" of the curves becomes most marked in fields above 15 volts/cm, and it is above this field strength that the experimentally determined frequencies begin to deviate from the theoretical values appreciably, as may be seen from Fig. 5.

Since the terminal energy of the electrons exceeds that of the molecules by an amount depending on the voltage, the electrons moving under the action of an alternating sine wave field will not reach their terminal energy, but there will be a phase lag between this energy and the applied voltage. This might act in such a way as to change the mobility. If, however, the distance the electrons must travel before the terminal energy is reached is small, the lag would be small. That this distance is small has been shown by Compton[4] who computed that at atmospheric pressure the electrons gain 99 per cent of their terminal energy within .015 cm from the starting point, assuming that they start from rest. As may be seen from Fig. 2, the mobilities determined with a plate distance of 2.4 cm (values represented by crosses) check the values obtained with a plate distance of 1.55 cm to within the limit of experimental error so that any effect due to a non-steady state is slight.

In conclusion, the writer wishes to express his thanks to Professor Compton for the use of the manuscript of the paper referred to above.

UNIVERSITY OF WISCONSIN,
MADISON, WISCONSIN,
August 8, 1923.

THE IONIZATION OF CAESIUM VAPOR AND THE MOBILITY OF ELECTRONS IN THE BUNSEN FLAME

By B. T. Barnes

Abstract

Ionization of caesium vapor in a flame.—A flame with a cross section about 2 by 4 cm was formed above rows of quartz tubes fed with air and gas into which dilute Cs salt solution was automatically sprayed. The *conductivity c* as a function of concentration k of the solution was determined from the potential gradient measured by means of platinum test wires and from the current to the central disk of a guarded anode. The *concentration of vapor m* in the flame as a function of k was computed from the velocity of the flame, measured stroboscopically, and from the amount of solution sprayed. The *black body temperature* was measured by introducing NaCl vapor and matching the emission against that of a tungsten filament for the same wave-length, assuming the emission to be merely a temperature effect. These results enabled the *equilibrium constant of Saha K* to be computed for comparison with that given by Saha's equation, for it is shown that $mc/(c^2-1)=A+Bc$, where A and B are constants, and that $K=(B^2/A)RT$, when R is the gas constant. Actually the value of K was inserted in Saha's equation and it was solved for T. The two best values obtained agree within 25° with the observed temperatures; this result tends to confirm the Saha theory.

Mobility of electrons in the flame was also computed from the measurements made, the equation being $k_e=c_0'/BNe$, where c_0' is conductivity of the unsalted flame, and N is the Avogadro constant. The results range from 75 to 207 m/sec for 1 volt/cm, but the last two values, 75 and 82 were obtained under more constant conditions and are believed more reliable.

Introduction

THE experiments described in the following paper were done with the object of testing Saha's theory[1] of the ionization of metallic vapors at high temperatures. In a recent paper by A. A. Noyes and H. A. Wilson,[2] the constant K for the equilibrium between neutral metal atoms, positively charged atoms and electrons is calculated for several alkali metals from the earlier results by H. A. Wilson[3] on the electrical conductivity of flames containing alkali salt vapors. It is shown that the values of K so obtained do not differ greatly from those given by Saha's theory; but since the results used by Noyes and Wilson were not all derived from experiments on the same flame, it seemed desirable to make a complete set of the necessary measurements on a single flame so as to obtain a more satisfactory test of the theory.

[1] Saha, Phil. Mag. **40**, 478 (1920)
[2] Noyes and Wilson, Astrophys. J. **57**, 20-32 (Jan. 1923)
[3] Wilson, Phil. Trans. A **216**, 63-90 (1915)

Noyes and Wilson calculated the number of electrons per unit volume present in the flame from the specific conductivity and the electron mobility, but these quantities were not accurately known and were measured in different flames. In the experiments described below the concentrations of the caesium in the flame were determined directly so that the equilibrium constant K could be calculated from the relative conductivities at two known concentrations. The specific conductivity was also determined and from this the mobility of the electrons in the flame was calculated.

PART I. TEST OF SAHA'S EQUATION

1. Theoretical

According to Saha's theory, the equilibrium constant K, for the pressure in atmospheres, is given by the equation[2]

$$\log_{10} K = -5048\ V/T + (5/2)\log_{10} T - 6.56$$

where V is the ionization potential of the metal vapor and T is the absolute temperature. The constant K is equal to $p_+\ p_e/p$, where p_+ denotes the partial pressure of the positive metal ions, p_e that of the electrons, p that of the neutral metal atoms. The value of V for caesium is 3.873 volts.[4] Then, in the case of caesium,

$$\log_{10} K = -19{,}530/T + (5/2)\log_{10} T - 6.56 \qquad (1)$$

We may also calculate K from the electrical conductivities of the flame in the following manner. Since the fraction of the flame atoms ionized is very small, we may write the mass law equation for the ionization equilibrium of the flame atoms

$$p'_+\ p_e = \text{constant} = B \qquad (2)$$

where p'_+ is the partial pressure of positive flame atoms.

Now
$$p_e = p_+ + p'_+ \qquad (3)$$

Also, since the mobility of the positive ions is negligible compared with that of the electrons, the conductivity of the flame is almost entirely due to the electrons; or

$$p_e = Ac \qquad (4)$$

where A is a constant, and c the specific conductivity.

Let m be the number of mols of metal vapor per cc in the flame. Then the total pressure of metal atoms is

$$p + p_+ = mRT \qquad (5)$$

where R is the molar gas constant.

From the definition of K and (5) we get

$$p_+ = KmRT/(K + p_e) \qquad (6)$$

[4] Bull. Nat. Res. Coun. **2**, 168 (1921)

Combining (2), (3) and (6) and then substituting for p_e from (4) we get

$$Ac = KmRT/(K+Ac) + B/Ac \tag{7}$$

Letting $c=1$ when $m=0$, (7) gives $A=B/A$; whence, on simplifying, (7) reduces to

$$\frac{mc}{c^2-1} = \frac{A}{KRT}(K+Ac)$$

Putting $m=Gk$, where G constant and $k=$concentration, in mols per liter, of the solution sprayed,

$$kc/(c^2-1) = b+ac \tag{8}$$

where $b=A/GRT$ and $a=A^2/KGRT$; consequently

$$K=(b^2/a)\,GRT \tag{9}$$

By measuring the conductivity c of the flame for different concentrations k of the caesium solution sprayed into it, the values of the constants b and a were obtained from Eq. (8). The ratio G was determined experimentally; then knowing b, a, and G, we can evaluate K by Eq. (9) on substituting the experimentally determined value of T. Instead of this we will proceed as follows:

From (9)

$$\log_{10} K = \log_{10}(b^2G/a) + \log_{10} R + \log_{10} T \tag{10}$$

Equating (10) to (1),

$$(3/2)\log_{10} T - 19{,}530/T = \log_{10}(b^2G/a) + 8.47 \tag{11}$$

This expression we can solve by trial for T and compare with the experimental value in order to test the validity of the Saha equation (1) by means of which it was obtained.

2. *Measurement of the Temperature of the Flame*

Since any thermo-couple inserted in a flame is at a lower temperature than the flame itself on account of conduction of heat along the wires and radiation, some method of determining the actual flame temperature had to be devised in order to test Saha's theory. Let us assume the emission of light by a sodium flame is a pure temperature effect. If this is the case a flame sufficiently thick should emit sodium light of the same intensity as is found in this region of the spectrum of a "black body" at the temperature of the flame. To be sure, Paschen[5] found the light emitted from a sodium flame was twice as intense as the same spectral region given by a black body in the flame; but this can be explained by the black body being cooler than the flame. For light of 5890 A, a black body at 1890° K gives about half the intensity emitted at 2000°. Thus if the black body was 110° cooler than the flame, as was probably the case, Paschen's

[5] R. W. Wood, Phys. Optics, p. 595

experiment agrees with our assumption that sodium light emission is a pure temperature effect.

This hypothesis was used in the following way. Light from a tungsten filament was passed through a flame into which a saturated solution of "hypo" was being sprayed, and then was focused upon the slit of a spectroscope. When the intensity emitted by the filament in the region around 5890 A is equal to the intensity from a black body at the temperature of the flame, the absorption of the *D*-lines should be exactly compensated by the emission from the flame. If the "black body temperature" of the filament is greater than the flame temperature, the *D*-lines are reversed; if less, the sodium light is more intense than the surrounding spectrum. Consequently we have only to determine the filament current necessary to make the *D*-lines have exactly the same intensity as the surrounding spectral region, and then to measure the "black body temperature" corresponding to this filament intensity. This was done by removing the flame and replacing the spectroscope by a Lesbole optical pyrometer. Since the latter used light of 6500 A, direct measurement of the intensity emitted by the flame was not possible. However, the "black body temperature" measured for the filament in the manner described above is the actual flame temperature, if sodium light emission is a pure temperature effect.

On account of variations in the gas supply it was impossible to use the same flame temperature in all the conductivity measurements. During any one set of observations the temperature was kept constant by frequent measurement with a thermo-couple, adjusting the gas supply when necessary. Each of the five sets of results given at the end of this paper is for a different thermo-couple voltage, the five values ranging from 145.5 to 153 scale divisions. The black body temperatures corresponding to various voltages were measured and then a calibration curve was plotted giving the flame temperature corresponding to any thermo-couple voltage within this range.

3. Measurement of the Specific Conductivity

The sprayer, burner, and flame used are shown in cross-section in Fig. 1. The burner *B* consisted of three rows of quartz tubes of 5.5 mm diameter set very close together in the top of a small brass box. It was designed after preliminary experiments with other types and gave a uniform flame about 2 cm wide and 4 cm long. The anode *A* and cathode *C* were at opposite ends of the flame *F*. The sprayer *S* filled the gas and air mixture passing through it with a fine spray of the solution being used. To keep the gas pressure constant, the gas was passed through a regulator con-

sisting of a bell jar suspended from a balance into a vessel of water and arranged so as to close the gas inlet when an increase in pressure caused it to rise. Two such regulators in series proved more effective than one; and a micrometer valve between the second and the sprayer proved useful in adjusting the flame to any desired temperature.

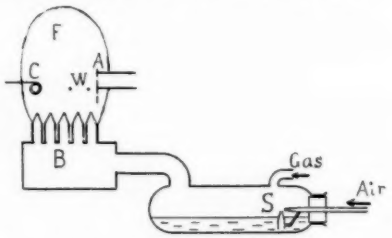


Fig. 1. Sprayer, burner, flame, electrodes and probe wires.

The compressed air was admitted by means of a needle valve and was passed through a glass wool plug to remove dust and water particles. A part was allowed to escape through a side tube dipping into mercury, in order to secure constant pressure and the remainder passed into a large bottle, to smooth out any oscillations, before entering the sprayer. To avoid contamination of the air in the room, some of which entered the flame, the flame gases were drawn up into a metal chimney, passing through a water cooler into the vacuum cleaner system.

The electrical connections for the specific resistance measurements are shown in Fig. 2. The potential gradient in the flame was measured by means of two fine platinum wires W, 1 cm apart, connected to a quadrant

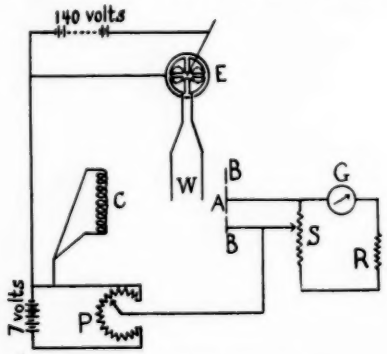


Fig. 2. Electrical connections.

electrometer E. For the anode, a platinum disk A, 8 mm in diameter, surrounded by a guard ring B, of 1 cm internal and 2 cm external diameter, was used. The current to the central disk was measured by means of a high sensitivity galvanometer G connected through the Ayrton

shunt S. In order to diminish damping, the series resistance R of 100,000 ohms was used. For the cathode C, a platinum disk 2 cm in diameter was first used but since this disk was always much colder than the flame it was later replaced by a horizontal spiral of platinum wire 2 cm in length. The potential applied to the electrodes was regulated by means of the potentiometer P. Hard rubber insulation was used throughout; and in addition paraffin coated blocks were placed under the electrometer and the 140-volt battery for charging the needle, to prevent the leakage across the ebonite supports commonly occurring in damp weather.

For parallel plane electrodes in a flame, previous experiments have shown that there is a region between the electrodes where the potential gradient is uniform as long as the current density is small. In this region the specific resistance should be independent of the current, since it depends only on the density of ions and the mobilities. However, preliminary experiments in the present research showed a decided increase in the value obtained for the specific resistance when the current was increased, as is shown in Fig. 3, in which curve A was obtained with the electrodes 4 cm apart and curve B with them 2 cm apart. Since

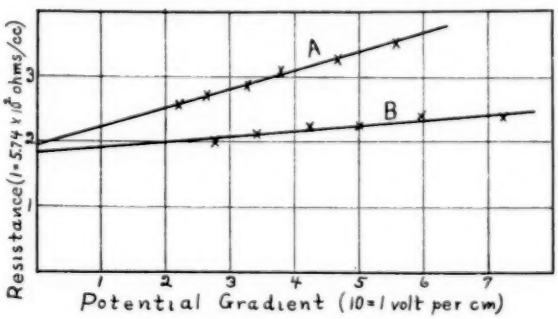


Fig. 3. Increase of specific resistance with gradient.
Electrodes 4 cm apart for Curve A; 2 cm apart for Curve B.

direct measurement showed the gradient was uniform for 22 volts or less between the electrodes, the reason for the increase is believed to be the following. The central disk of the anode was visibly colder than the guard ring since the latter shielded it from the flame. The ionization at the disk would, then, be less than at the surface of the guard ring. Consequently, the anode fall of potential would build up faster at the disk, diverting the current flow toward the guard ring. Thus the current density i from the disk, the quantity actually measured, would be less than that 1 cm in front of the disk, where the gradient was measured, and the value of X/i obtained would be too large.

As shown in Fig. 3, the increase of resistance with potential gradient was linear for small currents. Accordingly, in all subsequent measurements the resistance was determined for two different gradients, usually about 0.25 and 0.50 volts per cm. Then these two values were plotted and the value of the resistance for zero gradient obtained by drawing a straight line through the two points. This should be the true value.

For decreasing the electrode drop of potential, the use of K_2CO_3 on the cathode and sodium pyrophosphate on the anode was found unsatisfactory because the vapors diffused into the center of the flame. Consequently no salt was used on the anode, but lime, which gives little vapor and yet is fairly effective, was applied to the cathode by mixing with sealing wax. Though K_2CO_3 gives larger currents, the same specific resistance was found when lime was used.

In the actual measurements the electrodes were kept 4 cm apart, while the wires for measuring the gradient were about 0.7 and 1.7 cm from the anode.

4. Determination of the Constants a and b

For determining the constants b and a in Eq. (8), CsCl solutions of 2×10^{-5} and 10×10^{-5} normal concentration were sprayed. Fresh solutions were made up each day to reduce contamination due to dissolving of the glass of the containing flasks. The use of very dilute solutions is necessary in order to obtain b accurately, for since $b=kc/(c^2-1)-ac$, the possible error in the terms on the right becomes as large as b itself when these terms are large compared with b. For the conductivity of the flame without salt we take the value when distilled water is sprayed, in order to obtain the increase due to the dissolved salt alone when solutions are sprayed. This method gives the flame without salt the same water vapor content as when solutions are sprayed.

Fig. 4 shows the results of the specific resistance measurements in one of the best series of observations. Each point plotted is the average of three or four observations, and each curve was obtained after having filled the sprayer with a fresh solution. For distilled water and for 2×10^{-5} normal solutions one unit on the resistance axis $=1.325(10^4)$ ohms per cc; for the 10^{-4} normal solutions one unit $=0.651(10^4)$ ohms per cc.

Let R_0 = specific resistance for zero potential gradient. Taking the average R_0 for each concentration, Fig. 4 gives

	Distilled water	$N\times2(10)^{-5}$	$N\times(10)^{-4}$
R_0(ohms/cc)$\times(10)^{-4}$:	40.2	25.2	8.92
c:	1	1.593	4.51
$[kc/(c^2-1)]\times(10)^5$:	—	2.07	2.33

Solving for a and b we get

$$a=(2.33-2.07)\ (10)^{-5}/(4.51-1.593)=.0895\ (10)^{-5}$$

$$b=(2.07-1.593\times.0895)\ (10)^{-5}=1.93\ (10)^{-5}$$

The values of b and a for each of the five series of observations were obtained in this manner.

5. *Determination of the Density of Salt in the Flame*

In order to determine the ionization constant K by Eq. (9) we must know the ratio G of the number of mols m of metal vapor per cc in the flame to the concentration k of the solution sprayed.

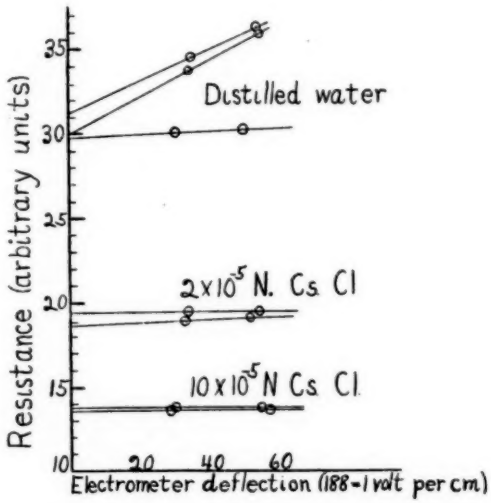


Fig. 4. Fourth set of conductivity measurements made on the flame.

If V = number of liters of solution entering flame per sec.; v = velocity of flame gases; and C = cross-sectional area of flame at height of electrodes, then $m=Vk/Cv$ and

$$G\equiv m/k=V/Cv \tag{12}$$

Since the flame was rectangular in cross-section, the area C could easily be measured. To determine the velocity v of the flame gases, puffs of vapor from a sprayer containing a saturated solution of NaCl were admitted to the flame through a rotating stopcock. These puffs emerged from a platinum tube opening just above the burner and became luminous as they were drawn up into the flame. When viewed through a stroboscopic wheel mounted on the shaft of the motor used to turn the rotating stopcock, they appeared stationary. By means of two horizontal wires which could be moved up and down in front of the flame, the distance between successive puffs was measured and then, knowing the

number of puffs per second from measurement of the speed of the motor turning the stopcock, the upward velocity v of the flame gases was calculated. During the conductivity measurements two different air pressures had been used on the sprayer. The mean of a number of readings for each pressure gave

$v = 173$ cm per sec. with 3.5 cm Hg pressure;
$v = 185$ cm per sec. with 5.5 cm Hg pressure.

To determine the volume V of salt solution entering the flame per sec., 50 cc of a standard solution of NaCl was put into the sprayer and the latter was run for half an hour or longer. The volume which had entered the flame was then determined by titrating the remainder with $AgNO_3$ solution. Evaporation does not affect the result except in so far as it changes the concentration of the solution being sprayed. By weighing the sprayer and burner before and after spraying, the loss due to evaporation was found to be several times that due to spraying. The resulting change in concentration was corrected for, but since this correction was smaller than experimental errors (about 3 per cent) it need not be explained. Corrected for change in concentration, $V = 4.54\ (10)^{-4}$ cc per sec. for 5.5 cm air pressure and $2.46\ (10)^{-4}$ for 3.5 cm pressure.

Observation of the cross-section C of the flame on one occasion gave 6.8 and 10.8 cm^2 for pressures of 3.5 and 5.5. cm respectively. Substitution of these values, together with the values for v and V given above, in Eq. (12) gives for the ratio G the values $2.27(10)^{-10}$ and $2.09(10)^{-10}$ for pressures of 5.5 and 3.5 cm respectively. For later experiments the cross-sectional area was 8 cm^2 for a pressure of 3.5 cm, giving $1.78(10)^{-10}$ for G.

6. Results of the Test of Saha's Equation

For the conductivity measurements plotted in Fig. 4, the cross-section of the flame was 8 cm^2 and the pressure 3.5 cm so that the ratio G was equal to $1.78(10)^{-10}$. The constants b and a were calculated for these observations in section 4. Substituting these values in Eq. (11) and solving for T we get T (theor.) $= 2030°$ absolute. In this case T (exper.) obtained from readings of the thermo-couple voltmeter by using the calibration curve secured by the method given in section 2, was also 2030°. However the probable error in the latter is at least ten degrees.

In the table below we give the results of five series of such observations in chronological order. The first set was taken with 5.5 cm Hg air pressure on the sprayer; the remainder with 3.5 cm pressure.

TABLE I

	$G\times(10)^{10}$	$a\times(10)^{7}$	$b\times(10)^{5}$	T(theor.)	T(exper.)
I	2.27	9.2	1.28	1980°K	2010°K
II	2.09	8.3	1.05	1950	2015
III	2.09	6.9	0.536	1865	1975
IV	1.78	8.95	1.93	2030	2030
V	1.78	6.90	1.62	2025	2000

The results given in lines IV and V of the table are much more reliable than the earlier results. Numerous improvements were made in the apparatus and experimental methods as the research progressed. Just before the last two series of observations were begun, the ebonite supports were sandpapered to obtain a fresh insulating surface, their number was reduced to a minimum, and paraffin coated blocks were introduced in addition to the ebonite supports under the table on which the electrical instruments were placed. Also the temperature remained nearly constant during these measurements and repeated readings agreed fairly consistently. On the other hand, successive readings in the second and third series of conductivity measurements varied considerably on account of difficulty in keeping the temperature constant. During series IV and V the cross-section was measured at the same time as the conductivity was measured at the same time as the conductivity measurements were being made. This fact also points to these as the most reliable.

Since the temperature determined experimentally agrees quite well, in the best two series of measurements, with that calculated by combining Saha's equation for the ionization constant K with the expression for K derived from the flame conductivity equations, the results obtained show that Saha's equation holds good within the limits of error of the present experiments.

PART II. MOBILITY OF ELECTRONS IN THE FLAME

The mobility of electrons in the flame was found from the specific conductivity in the following way.

Let c' = specific conductivity of flame;

n = number of electrons per cc;

k_e = velocity of an electron due to one volt per cm.

Then $c'=k_e e n$, since the velocity of the positive ions is negligible compared with that of the electrons. Also the pressure due to the electrons is given by $p_e=nRT/N$, where N = Avogadro's constant, so that $k_e = c'/en = c'RT/eNp_e$.

As given in section I of Part I, $p_e = Ac$, where c is the conductivity of the flame in terms of that without salt as a unit. Designating the latter by c_0', we have $c = c'/c_0'$. Then

$$k_e = c_0' \, RT/AN_e = c_0'/bGNe \tag{13}$$

Since c_0', b, and G were all determined in verifying Saha's equation we have only to substitute the values found into Eq. (13) to get k_e. In Table II are given the results for the same five series of observations as recorded in Table I above.

TABLE II

	$G\times(10)^{10}$	$b\times(10)^{5}$	$c_0'(\text{ohms}^{-1}/cc),\times(10)^{6}$	$k_e(\text{cm}^2/\text{volt sec.})$
I	2.27	1.28	5.81	20,700
II	2.09	1.05	2.86	13,500
III	2.09	0.536	1.91	17,700
IV	1.78	1.93	2.49	7,500
V	1.78	1.62	2.27	8,200

The disparity between the results of the first series of observations and those of the last two series is hard to explain. However the rough preliminary experiments gave even higher values for k_e indicating that the experimental errors due to poor insulation and impurities in the spray tend to increase k_e. These errors were lessened later in the research. As was pointed out previously, there is reason to believe that the results of series IV and V are fairly accurate, and, therefore, that the electron mobility in a Bunsen flame is around 8,000 cm per sec.

In conclusion, I wish to express my thanks to Dr. H. A. Wilson for many helpful suggestions during the course of the research and for assistance in preparing this paper, and to Rice Institute for facilities for research.

RICE INSTITUTE,
HOUSTON, TEXAS,
July 23, 1923.

THE ELECTRICAL PROPERTIES OF FLAMES CONTAINING SALT VAPORS FOR HIGH FREQUENCY ALTERNATING CURRENTS

By A. B. Bryan

Abstract

Dielectric properties of salted flames for high frequencies.—*The capacity C_1 and equivalent series resistance R_1* of a condenser consisting of two concentric platinum cylinders, 0.5 cm apart, in a flame containing K_2CO_3 vapor, are measured using a modification of the method of resistance variation. Values of C_1 and R_1 are obtained for voltages between 11.4 and 82.2 volts, for frequencies between 2×10^5 and 10^6 cycles per second and for salt concentrations up to 10 grams per liter of K_2CO_3 solution. The resistance is approximately independent of the field but decreases somewhat as frequency increases, and also as concentration increases. The capacity is half as great for 82 as for 11 volts, and for 10×10^5 as for 2×10^5 cycles, but increases five fold as the concentration increases from .1 to 10 gm per liter. The *apparent dielectric constant* varies from 1.2 for the unsalted flame to 9.1 for 10 gm per liter. For theoretical discussion see Wilson and Bryan, next article.

Introduction

THE electrical properties of flames containing salt vapors for high frequency alternating currents have previously been investigated by H. A. Wilson and E. Gold.[1] Their measurements were made with a capacity bridge having variable air condensers in three of its arms and a condenser formed of two concentric platinum cylinders in a flame in the fourth arm. The damped high frequency current used was obtaind by discharging Leyden jars through a spark gap. An electrolytic detector was used to find the balance point. They were able to get an approximate balance with the capacity bridge, that is, a definite minimum deflection of the galvanometer which was connected in series with the electrolytic detector, and hence concluded that the flame condenser behaved approximately like a pure capacity. They measured the capacity of the flame condenser and its variation with voltage, frequency and concentration of salt in the flame.

In the present work a similar condenser consisting of concentric platinum cylinders in a flame was used, but the experimental method permitted the measurement of the variation of both the capacity and the equivalent series resistance of the flame condenser with frequency,

[1] Wilson and Gold, Phil. Mag. **11**, 497, 1906

with voltage and with concentration of the salt solution which was sprayed into the flame.

Experimental Method

The method which has frequently been used for measurements of capacity and equivalent resistance for solid dielectrics and which has also been used for liquid dielectrics,[2] involves the assumption that these quantities are independent of the current flowing through the condenser. It was found that this assumption was not justified in the present case of a condenser having a flame as dielectric since both the capacity and the equivalent series resistance decreased as the current through the condenser increased. Consequently it was necessary to use a somewhat different experimental method for the work with flames. The measuring

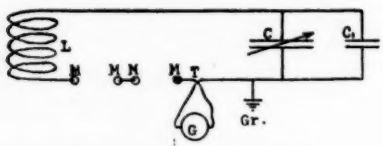


Fig. 1. Measuring circuit.

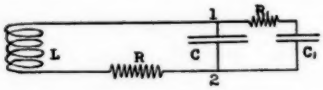


Fig. 2. Equivalent circuit.

circuit is shown in Fig. 1. The calibrated variable air condenser C and the flame condenser C_1 were permanently connected in parallel. The high frequency in the circuit was measured with the Western Electric vacuum thermo-couple T and the galvanometer G. The resistance of the circuit could be varied by connecting the mercury cups MM with resistance links of any desired value. Single layer coils of various sizes were used at L for coupling inductively to a high frequency oscillator using two $VT2$ tubes in parallel. The induced current in the measuring circuit could be varied by changing the distance between it and the oscillator; to facilitate this the entire measuring circuit was mounted on a rolling platform.

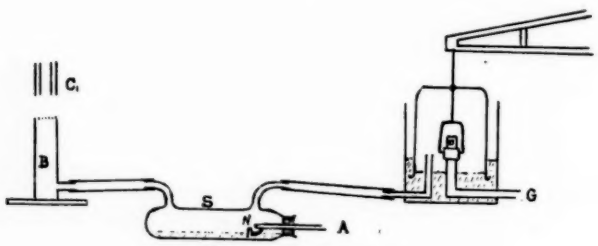


Fig. 3. Apparatus.

The arrangement of flame condenser, burner and sprayer is shown in Fig. 3. Gasoline gas passed into a special pressure regulator at G and

[2] A. B. Bryan, Phys. Rev. **22**, 399, 1923

then into the sprayer S. Air passed into the sprayer at A and produced at the nozzle N a fine spray of the solution with which the lower part of the sprayer was filled. The mixture of air, gas and spray passed into the burner B, which burner was 3.6 cm in diameter and gave a large steady flame completely surrounding the concentric platinum cylinders C_1 of the flame condenser and heating them to a bright red color. These platinum cylinders were each 6 cm long and 3 cm and 2 cm in diameter respectively. The outer cylinder was connected to the earthed side of the circuit.

The pressure of the air supply was always kept at 11.7 cm of mercury. A platinum-rhodium thermo-couple was fixed permanently in the flame 3 cm above the cylinders and was connected to a milli-voltmeter. The gas pressure was each time adjusted until this milli-voltmeter showed a certain deflection. In this way approximately the same flame may be reproduced from day to day. It is not possible to reproduce the same flame by merely using the same gas and air pressures at all times because of variations in the quality of the gas.

The procedure in making a measurement with this apparatus is as follows. When the desired flame has been secured the mercury cups MM are closed with links of zero resistance and the circuit is tuned to resonance with the oscillator by choosing the proper coil at L and by varying the capacity of the air condenser C. The current at resonance is made to have any desired value by changing the distance between the oscillator and measuring circuit. The resonance current is noted and then the flame is extinguished. This decreases the capacity of C_1 so that C must now be increased by an amount δC to restore resonance. Without the flame the resonance current will be larger than before because there was formerly a power loss in the flame condenser which became zero when the flame was extinguished. Accordingly a resistance δR is inserted which will bring the current down to the same value as when the flame was burning. It is difficult to design a continuously variable high frequency resistance and so the exact value of δR is obtained as follows. Various resistance links are put in at the mercury cups and the corresponding currents noted. The resistances are plotted as functions of the reciprocal of the current and from the resulting straight line, the exact value of δR for any current may be read. Each of the resistance links used was made of a piece of small straight manganin wire imbedded in paraffin in a glass tube, as suggested by Schott.[3] The diameter of the wire was always sufficiently small so that its resistance could without serious error be assumed to be the same at radio frequencies as for direct current.

[3] Schott, Jahrb. d. drahtl. Telegr. **18,** 103, 1921

In order to compare the experimental results with a theory of the behavior of the flame condenser it is convenient to assume that the flame condenser is equivalent to a pure capacity C_1 and a pure resistance R_1 in series. The complete measuring circuit may then be represented as in Fig. 2 and the values of C_1 and R_1 may be calculated from the measured values of δR, δC, C and the frequency.

It may be shown that in such a circuit,

$$\frac{E}{I}=\sqrt{\left[R+\frac{R_1C_1^2}{R_1^2\omega^2C_1^2C^2+(C+C_1)^2}\right]^2+\left[\omega L-\frac{R_1^2\omega^2C_1^2C+C+C_1}{\omega[R_1^2\omega^2C_1^2C^2+(C+C_1)^2]}\right]^2} \quad (1)$$

Where I = effective current through R;
E = effective induced e.m.f. in L;
$\omega = 2\pi \times$ frequency.

This gives the general relation between I and E in the measuring circuit. But in the experiments the measuring circuit was always in resonance with the oscillator and so it is necessary to see what Eq. (1) becomes at resonance, that is, when I reaches its maximum value as C is varied. If the quantity under the radical is differentiated with respect to C and the result set equal to zero we get

$$\omega L=\frac{A}{\omega B}\left[1-\frac{2R_1C_1^2\omega^2(RB+R_1C_1^2)}{B(R_1^2\omega^2C_1^2+1)-2A^2}\right]$$

Where $A=R_1^2\omega^2C_1^2C+C+C_1$; $B=R_1^2\omega^2C_1^2C^2+(C+C_1)^2$

The second term of the expression in parentheses may be shown to be small compared with unity by substituting known values. In one case it is equal to 0.00025, using the data obtained at 82.2 volts and 600,000 cycles with a solution of 1.0 gram of K_2CO_3 per liter as given in Table 2. (The value of R is not known exactly but from data previously obtained it is about 10 ohms.) If this small term is neglected we have approximately

$$\omega L=\frac{A}{\omega B}=\frac{R_1^2C_1^2\omega^2C+C+C_1}{\omega[R_1^2\omega^2C_1^2C^2+(C+C_1)^2]} \quad (2)$$

and if this value of ωL is substituted in Eq. (1)

$$\frac{E}{I}=R+\frac{R_1C_1^2}{R_1^2\omega^2C_1^2C^2+(C+C_1)^2} \quad (3)$$

When the flame is extinguished the measuring circuit becomes merely a circuit with inductance, capacity and resistance in series and,

$$\frac{E}{I}=\sqrt{(R+\delta R)^2+(\omega L-1/\omega C')^2}$$

where $C'=C+\delta C+C_0$; C_0 = capacity of the platinum cylinders in air.

At resonance, $$\omega L = 1/\omega C' \quad (4)$$

$$\frac{E}{I} = R + \delta R \quad (5)$$

I, E, R, ω and L are the same without the flame as with it and so by comparing Eqs. (2) and (3) with (4) and (5) we get,

$$\frac{1}{\omega C'} = \frac{R_1^2\omega^2 C_1^2 C + C + C_1}{\omega[R_1^2\omega^2 C_1^2 C^2 + (C + C_1)^2]}$$

$$\delta R = \frac{R_1 C_1^2}{R_1^2\omega^2 C_1^2 C^2 + (C + C_1)^2}$$

From these two equations we obtain

$$R_1 = \delta R\ (1 + C/C_1)\ (1 + C/\Delta)$$

$$C_1 = \Delta \frac{1 + \omega^2 C^2 \delta R^2 \left(1 + \frac{C}{\Delta}\right)^2}{1 - \omega^2 \Delta C\ \delta R^2 \left(1 + \frac{C}{\Delta}\right)^2}$$

where $$\Delta = C' - C = \delta C + C_0$$

The capacity C_1 and the equivalent series resistance R_1 of the flame condenser are thus expressed in terms of measurable quantities and may be determined for any frequency, concentration of salt solution or voltage across the condenser.

The voltage across the condenser is the potential difference E_1 between points (1) and (2), Fig. 2, and is equal to $I_1/\omega C$, where I_1 is the current through C. But $I_1 = I$ approximately, since C is large compared with C_1 and so $E_1 = I/\omega C$ approximately. I, ω, and C are measured and so E_1, the effective voltage across the flame condenser, may be calculated from this equation. The maximum value V_0 which is recorded in the tables is equal to $E_1\sqrt{2}$. I may be so changed by varying the coupling between measuring circuit and oscillator as to keep E_1 constant when ω and C change or to change E_1 while ω and C remain constant.

Results

The results obtained are presented in Table I. Voltage, frequency f and concentration of salt solution are successively varied and the resulting changes in R_1 and C_1 observed. All results were obtained with potassium carbonate solutions the concentrations K of which are given in grams per liter. All potential differences are in volts, resistances in ohms and capacities in micro-micro-farads. The capacity in air of the platinum cylinders, calculated from their dimensions, is $C_0 = 8.23\ \mu\mu f$. The apparent dielectric constant is not tabulated but is equal in any case to C_1/C_0.

TABLE I

V_0	f	K	C	δC	δR	C_1	R_1
82.2	6×10^5	1.0	630	4.0	1.71	12.8	4500
66.7	6	1.0	630	4.8	1.99	13.75	4580
47.5	6	1.0	629	6.2	2.44	15.4	4550
26.9	6	1.0	628	7.5	4.2	18.4	6060
11.4	6	1.0	621	12.6	6.35	25.3	5000
21.15	10	1.0	207	10.1	21.07	20.6	2860
21.15	8	1.0	161	10.7	42.75	21.4	3470
21.15	6	1.0	307	12.8	18.4	23.5	4030
21.15	4	1.0	1420	18.0	1.73	29.4	4700
21.15	2	1.0	870	30.0	7.65	39.8	4150
11.1	6	0	636	0.96	1.37	9.7	6420
11.1	6	0.1	631	5.15	3.61	15.7	7130
11.1	6	1.0	623	12.3	6.42	25.3	5170
11.1	6	5.0	614	21.6	10.95	39.3	3930
11.1	6	10.0	614	22.0	23.35	75.2	4560

A theory of the behavior of a flame condenser for high frequency alternating currents has been given by Wilson and Gold.[1] A modification of this theory, with which the above results are in good agreement, is given in the paper which follows this one.

In conclusion I wish to express my sincere thanks to Professor H. A. Wilson for his valuable advice and kindly interest in this work.

PHYSICAL LABORATORY,
THE RICE INSTITUTE, HOUSTON, TEXAS,
September 1, 1923.

THE CONDUCTIVITY OF FLAMES FOR RAPIDLY ALTERNATING CURRENTS

By H. A. Wilson and A. B. Bryan

Abstract

Conductivity of salted flames for high frequency currents.—*Theory.* Expressions are developed for the capacity and series resistance of a parallel plate condenser in a flame, which enable certain constants to be computed from the measurements of Bryan given in the preceding paper. *Calculated density of positive ions* increases from .03 e.s.u. in the unsalted flame to 1.5 e.s.u. for the flame sprayed with a solution of 10 gm/liter K_2CO_3. For the smaller concentration, it varies about as the square root of the potassium present, as was to be expected. The variation with frequency and electric field is slight. *Calculated mobility of electrons* is found to diminish from 200 to 60 m/sec. per volt/cm as r.m.s. field increases from 9 to 64 volts/cm, in good agreement with values found by Loeb for electrons in pure nitrogen at room temperatures. It decreases markedly as the concentration of potassium increases.

THE experiments described in the preceding paper by A. B. Bryan show that a pair of electrodes in a Bunsen flame containing an alkali metal vapor have a capacity for rapidly alternating currents depending on the amount of vapor present, on the frequency of the electrical oscillations and on the potential difference. There is also a resistance which depends on the same variables. The theory of the conductivity has been discussed previously by H. A. Wilson and E. Gold[1] who also made measurements of the capacity but did not determine the resistance. In this paper the results obtained by A. B. Bryan are compared with the theory of Wilson and Gold, which has been extended so as to include the resistance as well as the capacity. The theory enables the number of ions present in the flame and also the mobility of the negative ions to be calculated from the experimental results.

Consider two parallel metal plates with the space between them filled with a strongly ionized gas, the negative ions being free electrons. The mobility of free electrons is very large compared with that of positive ions so that the motion of the positive ions due to an alternating electric field between the plates is negligible compared with that of the electrons. The electrons will vibrate in the alternating field and those near the plates will strike them and so be removed from the gas. We shall therefore have a layer of electrons vibrating between the plates. Inside this layer

[1] Wilson and Gold, Phil. Mag. **11**, 497, 1906

there will be as much positive electricity as negative and outside it only positive ions. If D is the distance between the plates and $D-d$ the thickness of the layer of negative electrons then, following Wilson and Gold, it is easy to show that

$$V=2\pi\rho d\ (2t_1-d)-XD$$

where V is the potential difference between the plates, X the electric intensity, supposed uniform, in the layer of electrons, ρ the charge per cc on the positive ions and t_1 the distance of the layer of electrons from the negative plate. Let $x=t_1-d/2$ so that

$$V=4\pi\rho dx-XD \tag{1}$$

x is then the displacement of the layer of electrons from its mean position and $\frac{1}{2}d$ is the amplitude of the vibration. We shall suppose that the inertia of the electrons may be neglected so that

$$dx/dt=-kX \tag{2}$$

where k is the electron mobility or velocity for unit electric field. The current density j in the layer of electrons is given by

$$j=-\rho\frac{dx}{dt}+\frac{1}{4\pi}\frac{dX}{dt} \tag{3}$$

since it is the sum of the convection and displacement currents.

These equations may be solved in the case of an alternating potential difference, given by $V=V_0\epsilon^{ipt}$ by assuming $x=x_0\epsilon^{ipt}$, $X=X_0\epsilon^{ipt}$ and $j=j_0\epsilon^{ipt}$. Substituting these values we get

$$V_0=4\pi\rho dx_0-X_0D \tag{4}$$

$$x_0ip=-kX_0 \tag{5}$$

$$j_0=-\rho x_0ip+X_0ip/4\pi \tag{6}$$

and we also have the condition that the amplitude of x is to be put equal to $\frac{1}{2}d$.

Eliminating x_0 and X_0 we obtain

$$j=\frac{V_0p\epsilon^{i(pt+\gamma)}}{4\pi}\sqrt{\frac{(4\pi\rho k)^2+p^2}{(4\pi\rho kd)^2+p^2D^2}} \tag{7}$$

$$\tan\gamma=\frac{(4\pi\rho k)^2d+p^2D}{4\pi\rho kp(D-d)} \tag{8}$$

$$d=(1/4\pi\rho k)[\sqrt{(8\pi V_0\rho k^2)^2+\tfrac{1}{4}(pD)^4}-\tfrac{1}{2}(pD)^2]^{\frac{1}{2}} \tag{9}$$

and

$$X=V_0\ p\ \epsilon^{i(pt+\alpha)}/\sqrt{p^2D^2+(4\pi\rho kd)^2}$$

where $\tan\alpha=4\pi\rho kd/pD$.

When an alternating potential difference $V_0\epsilon^{ipt}$ acts on a condenser of capacity C_1 with a resistance R_1 in series, the current is equal to

$$V_0\epsilon^{i(pt+\gamma)}/\sqrt{R_1^2+(pC_1)^{-2}}$$

where $\tan\gamma=(R_1pC_1)^{-1}$

Comparing these expressions with those previously found for the strongly ionized gas, noting that the total current $c=ja$, where a is the area of the electrodes in the gas, we get

$$C_1=\frac{a}{4\pi}\left[\frac{(4\pi\rho k)^2+p^2}{(4\pi\rho k)^2 d+p^2 D}\right] \tag{10}$$

$$R_1=\frac{16\pi^2\rho k(D-d)}{a\left[(4\pi\rho k)^2+p^2\right]} \tag{11}$$

Eqs. (9) (10) and (11) enable ρ, k, d and X to be calculated from the experimentally determined values of C_1 and R_1 found by A. B. Bryan as described in the preceding paper.

Eqs. (10) and (11) on putting $C_0=a/4\pi D$ give

$$(C_1-C_0)/R_1C_1C_0=4\pi\rho k \tag{12}$$

Putting $4\pi\rho k=z$ and $d/D=F$, (10) then gives

$$F=(C_0/C_1)\ (1+p^2/z^2)-p^2/z^2 \tag{13}$$

and (9) gives

$$k=(FD^2/2V_0)\ \sqrt{p^2+z^2F^2}$$

The value of ρ is then given by $\rho=z/4\pi k$. The following table gives the values of F, k, ρ and $\overline{X}$, the r.m.s. value of X calculated from Bryan's values of C_1 and R_1. C_0 was equal to 8.23 $\mu\mu$f or 7.4 cm and D was 0.5 cm.

TABLE I

C_1 ($\mu\mu f$)	R_1 (ohms)	V_0 (volts)	f (per sec.)	K (g/liter)	F (d/D)	k $\left(\frac{\text{cm/sec}}{\text{volt/cm}}\right)$	ρ $\left(\frac{\text{e.s.u.}}{\text{cc}}\right)$	X $\left(\frac{\text{volts}}{\text{cm}}\right)$
12.8	4500	82.2	6×10^5	1.0	.589	6100	0.42	64.
13.75	4580	66.7	6×10^5	1.0	.548	7300	0.39	51.
15.4	4550	47.5	6×10^5	1.0	.492	9300	0.36	35.
18.4	6060	26.9	6×10^5	1.0	.382	10000	0.29	25.
25.3	5000	11.4	6×10^5	1.0	.290	19300	0.25	9.4
20.6	2860	21.15	10×10^5	1.0	363	24000	0.28	17.
21.4	3470	21.15	8×10^5	1.0	.352	19000	0.30	16.6
23.5	4030	21.15	6×10^5	1.0	.327	14400	0.36	15.2
29.4	4700	21.15	4×10^5	1.0	.267	8800	0.56	13.4
39.8	4150	21.15	2×10^5	1.0	.205	6000	1.04	7.6
9.7	6420	11.1	6×10^5	0	.60	28200	.028	14.1
15.7	7130	11.1	6×10^5	0.1	.423	24300	.089	11.6
25.3	5170	11.1	6×10^5	1.0	.287	19000	.221	10.0
39.3	3930	11.1	6×10^5	5.0	.191	12800	.50	10.0
75.2	4560	11.1	6×10^5	10.0	.087	4230	1.49	13.9

We should expect ρ to depend only on the amount of salt in the flame. Of the eleven values of ρ obtained while a solution containing one gram per liter of potassium carbonate was being sprayed into the flame, the majority do not differ much from about 0.35. The theory can be regarded only as a first approximation so that the variation of ρ is not greater than might have been expected. The theory is probably not

applicable unless the frequency is very large and so fails in the case of the two lowest frequencies of 2×10^5 and 4×10^5, for which the values of ρ are considerably greater than the mean value.

The last five rows in the table show the variation of ρ due to changing the concentration of the salt in the flame. ρ increases with the amount of salt as we should expect. With the smaller concentrations ρ increases about as the square root of the concentration but with the larger concentrations is more nearly proportional to the concentration. These results agree with the earlier measurements of the conductivity of K_2CO_3 in flames using steady currents.[2]

The relation between the r.m.s. field strength $\overline{X}$ and the mobility k is shown in Fig. 1, where the seven values of k obtained when spraying a K_2CO_3 solution containing one gram per liter are plotted. It appears that the mobility falls as $\overline{X}$ increases.

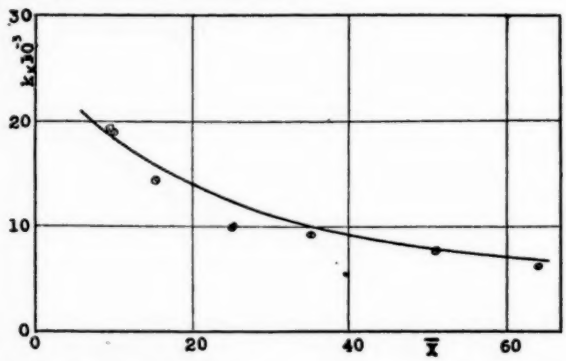


Fig. 1. Variation of mobility of flame electrons with r.m.s. field. Results of Bryan compared with values given by Loeb's formula.

Loeb[3] has found that the average mobility of electrons in pure nitrogen at 760 mm pressure is equal to $571000/(21+\overline{X})$. The curve drawn in Fig. 1 represents the values of the mobility given by Loeb's formula. The agreement between curve and points seems to indicate that the mobility of electrons in gases is not much affected by temperature at constant pressure.

We should expect the mean free path to be proportional to the absolute temperature and the velocity of agitation to vary as the square root of the absolute temperature, so that for small fields the mobility might be expected to vary as the square root of the absolute temperature at constant pressure. The agreement between Loeb's mobilities at the ordinary temperature and those found in the flame suggests that the increase in the

[2] Smithells, Dawson and Wilson, Phil. Trans. A **193**, 89, 1899
[3] Loeb, Phys. Rev. **19**, 24, 1922

velocity of agitation due to the field makes the velocity of agitation of the electrons nearly proportional to the absolute temperature so that the mobility is nearly independent of the temperature. However, since the calculation of the mobilities in the flame is based on an approximate theory of the conductivity and since flame experiments are subject to considerable errors, too much weight must not be given to this agreement.

In nitrogen at ordinary temperatures Townsend[4] found the electron velocity to be 5.2×10^5 cms per sec. when $X/p=0.25$, where p is the pressure in mm of mercury. Using Loeb's formula we can calculate the velocity of the electrons in the flame when $X/p=0.25$. Since $p=760$, we require the velocity when $X=190$. It comes out $2710\times190=5.15\times10^5$ cm per second and so agrees well with that found by Townsend.

The first measurements of the mobility of the negative ions in flames were made by H. A. Wilson[5] who got about 1000 cm per sec. for one volt per cm by a rough method only capable of giving the order of magnitude of k. Numerous other estimates have been made, among which is that of E. Gold[6] who got $k=12900$ for very small values of X. H. A. Wilson[7] found $k=2450$ from measurements of the Hall effect with X about 30 volts per cm. The high values for the mobility of the negative ions in flames indicate that they are free electrons, as was first suggested by Lenard.

The values of k found with different amounts of salt, given in the last five rows of the table, indicate that k diminishes as the amount of salt increases. This may be due to the electrons getting attached to potassium atoms during a part of their existence as negative ions, which is possible when X is small, as was true in all these cases.

The values of k found with different frequencies, given in the second five rows of the table, indicate that k diminishes with the frequency. This variation may be apparent rather than real, however, since the results for the two lowest frequencies are open to suspicion, as already mentioned and the other three do not show much change.

DEPARTMENT OF PHYSICS,
THE RICE INSTITUTE, HOUSTON, TEXAS,
September 1, 1923.

[4] Townsend, Phil. Mag. **42**, 873, 1921
[5] Wilson, Phil. Trans. A **192**, 499, 1899
[6] Gold, Proc. Roy. Soc. A **79**, 61, 1907
[7] Wilson, Phys. Rev. **3**, 375, 1914

SOME MOLECULAR ABSORPTION SPECTRA OF HYDROGEN CYANIDE

By E. F. Barker

Abstract

Infra-red absorption maxima due to hydrogen cyanide.—Four of the infra-red bands due to HCN have been associated by Kratzer as fundamental and successive harmonics. Of these, the fundamental at 14μ can not be reached with the grating spectrometer at present, but the three others have been examined with specially ruled gratings giving higher dispersion than was previously employed. Two maxima, very nearly symmetrical and of equal intensity, appear at 6.94μ and 7.23μ. Three others, less intense and unsymmetrical, are found at 4.756μ, 4.723μ and 4.79μ. There is a sharp single maximum at 3.564μ. The forms of the curves and the separations between maxima raise some doubts as to whether the higher frequency bands are second and third harmonics of the one at 14μ. If they are, it appears that the intensity of the zero branch increases with the order of the vibrational transition. The band series could not be resolved. Estimates of the *moment of inertia of the molecule* from various bands do not agree. The measurements at 7μ give about 13.2×10^{-40}, which is five times that of HCl.

THE chemical behavior of cyanogen is in many respects similar to that of chlorine. The CN radical exhibits a stability which suggests an unusually intimate binding between its two atoms, and, since its electron affinity is large, readily forms negative ions. These are found in cyanide crystals having structures similar to those of the halides. Langmuir[1] has attributed to them a configuration identical with that of the nitrogen and carbon monoxide molecules. The ionization potential of HCN falls in line with those of the halogen acids, indicating a bond between the H^+ ion and the CN^- group slightly more intense than that between H^+ and Cl^-.[2] These facts at once suggest the possibility of a relatively simple vibration-rotation spectrum for hydrogen cyanide, and of a small molecular moment of inertia.

Five bands in the spectrum of HCN have been observed by Burmeister[3] between 3μ and 15μ. Of these, the one having the shortest wave-length, 3.04μ, is very narrow and intense, apparently corresponding to the fundamental of one characteristic vibration of the molecule. The two bands of longest wave-length, near 7μ and 14μ, are also intense, but appear as

[1] Langmuir, J. Am. Chem. Soc. **41**, 905, 1919
[2] Knipping, Zeits. f. Phys. **7**, 328, 1921
[3] Burmeister, D. Phys. Ges. Verh. **15**, 589, 1913

broad double maxima. The absorption at the two intermediate maxima, 4.77μ and 3.57μ, is relatively weak and Burmeister was in doubt as to whether these bands should be attributed to HCN. Kratzer,[4] however, concluded from the numerical relation between frequencies that they are the second and third harmonics of the 14μ band, thus giving to them a peculiar interest, since few harmonics higher than the first have been observed in infra-red spectra.[5] In the present investigation these bands have been found in the spectrum of pure HCN, always with about the same intensities, and hence are probably not to be attributed to impurities as Burmeister supposed. The percentages of absorption at the two maxima are not in the ratio which would be suggested by Kratzer's hypothesis, although total absorptions may be; neither are the various bands similar in form as would be expected for successive harmonics.

The apparatus employed has previously been described.[6] Three gratings were selected for concentration of the energy in the regions 3μ to 4μ, 4μ to 6μ, and 6μ to 8μ. Unfortunately it has not yet been possible to make observations beyond 12μ due to the small amount of such energy in the incident beam and the increasing absorption of the rock salt prism. Hence the pair of absorption maxima described by Burmeister at 13μ and 14μ have not been re-examined. The form of these maxima is of particular significance if they do constitute the fundamental, and it is hoped that later they may be reached. HCN gas was generated by introducing below the surface of H_2SO_4 a solution of NaCN in water. The gas was collected by condensation in a trap provided with two stopcocks, cooled to nearly 0°C, and each sample to be examined was distilled over to the absorption cell by dipping the trap into a water bath at about 35° C. A considerable amount of gas was allowed to pass through the cell before closing the stopcocks, so that the enclosure was very thoroughly washed out, and while in use the cell itself was heated slightly to prevent condensation of the gas.

The region of 7μ was examined with a grating having 2880 lines per inch, and giving deflections of about 20 mm. The absorption cell was 4 cm long, with very thin mica windows which were protected from pressure variations by having the enclosure connected to a deflated rubber balloon. This cell was moved alternately into and out of the beam. Observations in this region are rendered difficult by the absorption of atmospheric water vapor which presents many sharp maxima where deflections taken either with or without the absorbing cell are reduced

[4] Kratzer, Zeits. f. Phys. **3**, 289, 1920
[5] Hettner, Zeits. f. Phys. **1**, 345, 1920
[6] Barker, Astrophys. J. **55**, 391, 1922

below the limit permissible for precise measurement.[7] Between the water vapor lines no difficulty is encountered. The form of the curve is shown in the figure, Curve I. There are two maxima, very nearly

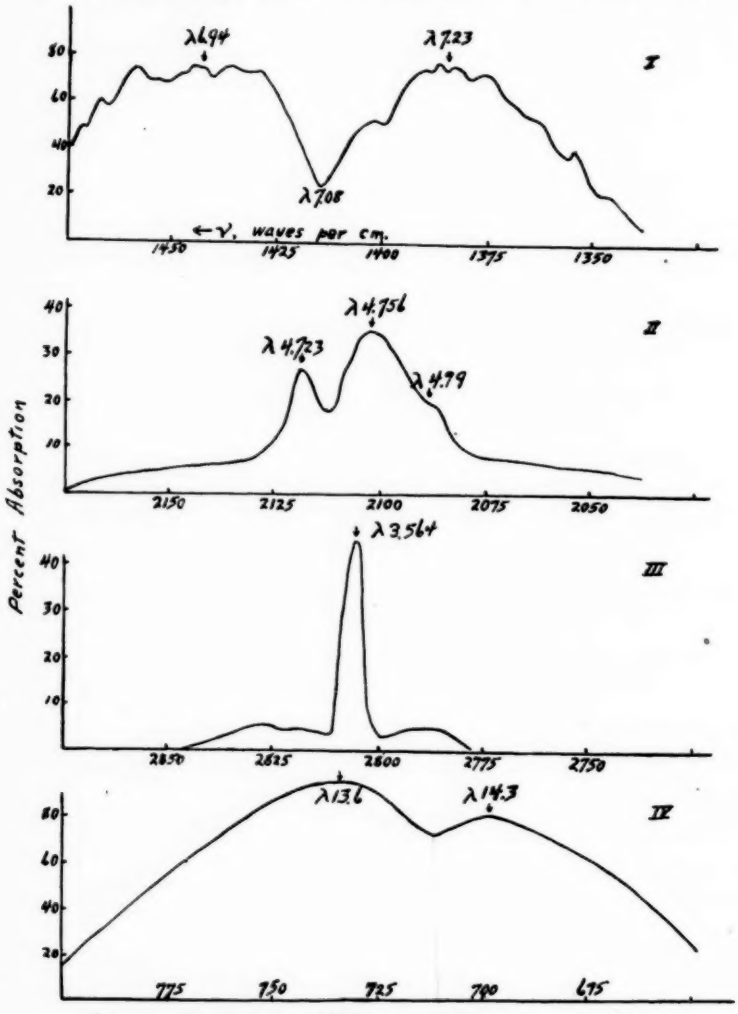


Fig. 1. Absorption of HCN, atmospheric pressure, 30°C.

Curve I. 4 cm cell, 2880 line grating, slit 0.01μ.
Curve II. 30 cm cell, 2880 line grating, slit 0.01μ.
Curve III. 30 cm cell, 7200 line grating, slit 0.003μ.
Curve IV. From Burmeister's data, slit 0.3μ.

symmetrical, with nearly equal intensities, separated by a sharp minimum. Their wave-lengths are about 6.94μ and 7.23μ, the frequency

[7] Sleator and Phelps, Phys. Rev. **19**, 394, 1922

difference between them being 58 waves per cm. Measurements have not been extended in either direction to the regions of very low absorption, on the low frequency side because the energy of the beam falls off rapidly, and on the high frequency side because of increasing intensity of the water vapor absorption. If this be considered as a typical doublet with a nearly symmetrical though unresolved band series the corresponding moment of inertia is approximately 13.2×10^{-40}, and the mean separation between successive members should be $h/4\pi^2I = 12.5\times10^{10}$, or 4.2 waves per cm. As this is about twice the slit width, such a series would be barely observable. In this connection it is of interest to note that Burmeister's measurements on the fundamental band lead to a wave number difference between maxima of 37.5 per cm and consequently to a moment of inertia nearly four times as large as the one obtained above. Considered as simple bands, neither the 14μ nor the 7μ regions show any evidence of a zero branch, each absorption curve being nearly symmetrical with respect to a deep notch. This would indicate an axial symmetry in the molecule.

The region of 4.7μ shows a very much less intense absorption and could scarcely be located with the 4 cm cell. The second curve represents the absorption in 30 cm of saturated vapor at about 30° C and atmospheric pressure. It is plotted to the same frequency scale as the previous one, with about the same dispersion. These measurements are derived from two energy curves each taken through the cell and each the average of several independent sets of data. One corresponds to transmission through the air filled cell and the other to transmission through HCN. This procedure was employed because of the difficulty of swinging the long cell into and out of the optical path. As a consequence little precision can be claimed for the absolute values of the ordinates at those points where absorption is only a few per cent, but repeated observations indicate that the extended portions on either side of the maxima represent real absorption, with no noticeable maxima or minima. This background may perhaps be explained as the superposed effect of several weak bands somewhat displaced with respect to one another. The principal maximum appears at 4.756μ, with a second narrow one at 4.723μ, and apparently a third in the neighborhood of 4.79μ. From this curve it would be difficult to make an estimate of the value of the moment of inertia in the usual way, but obviously the wave number differences are not of the same magnitude as those exhibited at 7μ.

The absorption band near 3.6μ (Curve III) is also weak. Although it was observed with a 4 cm cell, the measurements are for the 30 cm HCN path. A single narrow and fairly intense maximum is found at 3.564μ,

with a low background on either side, most intense toward higher frequencies. There is here little to suggest the typical doublet. In fact a comparison of the three curves, with a fourth here reproduced from Burmeister's observations, at once raises a doubt as to whether or not the several transitions producing them can be intimately related. If the band represented in Curve III actually results from a vibrational transition from 0 to 4 as Kratzer suggests, it would seem that this is accompanied by rotational changes much less frequently than are the transitions of lower order. In other words this single maximum might be interpreted as the head of a zero branch or Q series, enhanced with respect to the other two associated series.

It has not been generally supposed that the probability of the occurrence of a zero branch changes appreciably from member to member in a series of bands. If, however, this be assumed as a possibility, the central maximum of Curve II would then represent a somewhat less intense zero branch, and the frequency differences between maxima of the positive and negative branches in each of Curves II, III and IV become approximately 35 waves per cm. Curve I could only be correlated with the others by assuming it to represent two complete bands instead of a single one, but neither of the corresponding frequencies would fit in Kratzer's equation relating harmonic vibrations.

The above observations were taken, and the necessary gratings ruled in the Physical Laboratory at the University of Michigan while the writer held a fellowship under the National Research Council.

NATIONAL RESEARCH FELLOWSHIP,
UNIVERSITY OF MICHIGAN,
September 1, 1923.

EFFECT OF A TRANSITION LAYER ON INTRINSIC POTENTIAL[1]

By P. V. Wells

Abstract

Frenkel's theory of the intrinsic potential is generalized to include the effect of a transition layer. The expression derived is $\varphi = 2\pi n \Sigma\ (E\overline{r^2\cos^2\theta})$, where n is the number of molecules per cc, and the summation is made of the mean square normal component distances of the electrons and nuclei from the center of mass of the molecule, one term for each electron and one for each nucleus, E being the corresponding charge. The intrinsic potential depends, then, on the average positions of the electrons and nuclei, but is independent of the form of the transition layer. For molecules with spherical symmetry, the expression reduces to that of Frenkel.

Introduction

IN VIEW of the improvements which Van der Waals introduced into Laplace's theory of surface phenomena by considering the effect of a transition layer, it occurred to me that Frenkel's theory[2] of the intrinsic electric potential of solid and liquid bodies might also be improved by similar considerations. Van der Waals simplified the transition layer by neglecting derivatives of the fourth and higher orders in the expansion for the local density, which seems hardly consistent with a transition layer of not many molecules; but I find this unnecessary. The assumption of a perfectly general transition layer does not introduce any complications in the derivation of the intrinsic potential, provided averages are taken before integrating, as in Frenkel's development. This shows that the simplicity of Frenkel's analysis is not due to his use of the abrupt surface of Laplace, but to the assumption of statistical spherical symmetry in the molecule.

In the following article, I have generalized Frenkel's theory to include any transition layer, at the same time making more evident than he did the statistical assumptions involved. The expression for the intrinsic potential reduces to that of Frenkel in the special case of spherical symmetry, and so leads to values which fall in the range given by experiment. But the general expression indicates clearly how the average

[1] Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

[2] J. Frenkel, Phil. Mag. **33**, 297-322, 1917

positions of the protons and electrons in the molecule must be assumed or known to predict the intrinsic potential.

While perhaps a more rigorous analysis would require the averages to be taken after the integration, not before, there can be little question that an electric potential exists quite independent of surface structure, an intrinsic potential truly characteristic of the internal structure of the body. Indeed, it appears that an intrinsic potential follows simply from the finite separation of the two electricities, and represents the potential energy of molecular configuration. Conversely if this be accepted, it furnishes additional evidence for the existence of a concentrated positive nucleus.

Effect of a Transition Layer on Intrinsic Potential

The fundamental notion that all bodies must possess a characteristic intrinsic potential, quite apart from their surface transition layers, seems to be a direct consequence of the finite separation of the two electricities. This potential energy of electrical configuration may evade our observation because natural surfaces are dumping grounds for all sorts of stray molecules which tend to lower the surface energy. We shall nevertheless attempt to show that this intrinsic potential is independent of the particular form of the transition layer for substances in a state of ideal purity.

Consider the plane boundary of a pure liquid or solid body of but one molecular species, in equilibrium with its vapor. In this plane our point of view will be molar, that is, we shall require statistical quantities only, such as the average number of atoms per square centimeter. But in the direction normal to the surface the quantities must be functions of the exact plane x in the transition layer. The number of molecules per cc is assumed to be $n_x \equiv f(x)$, a continuous function of x. The form of the function n_x determines the structure of the transition layer, as in the theory of Van der Waals. Then by Taylor's Series, the molecular density at $(x-r)$ is

$$n_{x-r} = n_x - r\frac{dn}{dx} + \frac{r^2}{2!}\frac{d^2n}{dx^2} - \frac{r^3}{3!}\frac{d^3n}{dx^3} \cdots \quad (1)$$

The molecules are never at rest, because of the incessant thermal agitation, so that the ideal continuous function (1) seems justified statistically, as an average both over the surface and in time. The motion of the mass centers of the molecules is small compared with the more pronounced oscillations of the atomic nuclei, and these are negligible compared with those of the electrons, so that the positions of the electrons and nuclei can be averaged as if these centers were at rest.

Let θ_1 be the angle at any instant between the radius r_1 from the center of mass of the molecule to a given electron or nucleus (marked by subscript 1) and the normal into the surface. This electron is in the plane x when the molecule is in the plane $(x-d_1)$, where $d_1=(r_1\cos\theta_1)$ is the normal component of the distance from the center of mass of the molecule, reckoning inwards. The average number of such electrons per unit volume at the plane x is given by

$$\sum p_1 n_{x-d_1}=n_x-\overline{d_1}\frac{dn}{dx}+\frac{\overline{d_1^2}}{2!}\frac{d^2n}{dx^2}\cdots\cdots \tag{2}$$

where p_1 is the chance of the electron having the position d_1 in any molecule, and $(\overline{d_1}\equiv\Sigma p_1 d_1)$, $(\overline{d_1^2}\equiv\Sigma p_1 d_1^2)$, etc. are the average values over a molecule. Summing over all the electrons of charge $-e$ each, and over the nuclei, weighting each by its number of unit charges in the molecule, the net local electric density ρ_x is

$$\rho_x=a_1\frac{dn}{dx}-a_2\frac{d^2n}{dx^2}+a_3\frac{d^3n}{dx^3}-a_4\frac{d^4n}{dx^4}\cdots\cdots \tag{3}$$

where

$$a_1\equiv e\,\Sigma\,(\bar{d}),\ a_2\equiv(e/2!)\,\Sigma\,(\overline{d^2}). \tag{4}$$

The term in n_x drops out because of the electric neutrality of the molecule.

The most restrictive assumption involved in (3) is that none of the averages a_1, a_2, a_3, etc., are functions of x. This means that the average orientation of the molecules in any one layer is the same as in any other, so that if they were all pointed one way over the surface, they would all point the same way along its normal across the transition layer. The statistics a_1, a_2, a_3, etc. can be used as parameters to define an equivalent ideal continuous distribution of electricity ρ_x, which is uniform over each layer dx and constant in time. Assuming the electric intensity E_x to be given by

$$dE_x/dx=4\pi\rho_x \tag{5}$$

in virtue of the symmetry, we obtain as the local electric intensity

$$E_x=4\pi\left[a_1 n_x-a_2\frac{dn}{dx}+a_3\frac{d^2n}{dx^2}-a_4\frac{d^3n}{dx^3}\cdots\cdots\right] \tag{6}$$

The constant of integration is zero because the intensity must vanish at infinite distance from the body. Integrating again, the expression for the local potential ϕ_x is

$$\phi_x=4\pi\left[-a_1\int_{-\infty}^{x}n_x dx+a_2 n_x-a_3\frac{dn}{dx}+a_4\frac{d^2n}{dx^2}\cdots\cdots\right] \tag{7}$$

the integral being taken across the transition layer into the surface. Now within the body the molecular density n is constant, so that the derivatives vanish. Moreover a_1 must vanish, for otherwise the integral increases without limit as we penetrate the body. This requires the

planes x to be equipotential surfaces. The electric intensity, Eq. (6), within the body is therefore zero, which is known to be the case experimentally. The intrinsic potential ϕ thus reduces to

$$\phi = 2\pi\, n \sum (E\, \overline{r^2 \cos^2\theta}) \tag{8}$$

where n is the number of molecules per cc, E the charge on electron or nucleus, and the summation is taken of the mean square normal component distances of the electrons and nuclei from the center of mass of the molecule, a term being added for each electron and one for each nucleus. Since dimensionally n is the reciprocal of a volume, the potential is the ratio of a charge to a length, as it should be. While a known orientation of the molecules enables us to evaluate the intrinsic potential, Eq. (8), no knowledge is necessary of the density function n_x in the transition layer.

For monatomic bodies Frenkel assumed spherical symmetry in the distribution of electrons about the nucleus. If we do this, for κ valence electrons at radius r,

$$\sum (\overline{r^2 \cos^2\theta}) = \kappa\, 2\pi r^4 \int_0^{\pi/4} \cos^2\theta \sin\theta d\theta / 2\pi r^2 \int_0^{\pi/4} \sin\theta d\theta = \tfrac{1}{3}\kappa r^2 \tag{9}$$

and our general Eq. (8) reduces to

$$\phi = \tfrac{2}{3}\pi\kappa n e r^2 \tag{10}$$

which is Frenkel's formula. The nucleus gives no contribution ($r=0$), and that of the completed inner electron rings is negligible, because of their small radii. From this it is seen that Frenkel's expression does not depend at all upon the discontinuous transition layer he assumed, but upon the assumed spherical symmetry of the atom.

Our Eq. (8) indicates clearly how the average positions of the protons and electrons in the molecule must be assumed or known to predict the intrinsic potential. In the present state of the theory of atomic structure models of complicated molecules are premature, and the agreement of Frenkel's simpler formula (10) with experiment is quite good enough. Frenkel goes on to explain the molecular pressure by the stress in the surface double-layer, and the surface-tension by its energy, always assuming the abrupt surface of Laplace. The theory of Van der Waals shows how Frenkel's theory could be improved, but such a development would require the assumption of a special transition layer, one much more abrupt than that of Van der Waals. Such special assumptions would have no effect upon the intrinsic potential, for it is independent of the form of the transition layer.

I wish to thank Prof. W. F. G. Swann for his criticisms and suggestions.

WASHINGTON, D. C.,
February 10, 1923.

THE MAGNETIC SUSCEPTIBILITY OF OXYGEN, HYDROGEN AND HELIUM

By A. P. Wills and L. G. Hector

Abstract

The method used was that of balancing the gas magnetically against an aqueous solution of nickel chloride. By varying the concentration of the solution it could be given a susceptibility approximately the same as that of the gas; then by varying the pressure of the gas or the temperature of the gas and solution, both could be given the same susceptibility. A *manometric balance of great sensitivity* enabled the observer to tell when the susceptibilities of the gas and the solution were the same. For both paramagnetic and diamagnetic gases, formulas are derived from which the susceptibility may be calculated from pressure and temperature observations on the gas when it is magnetically neutral against the solution. The volume susceptibility under a pressure of one atmosphere at the temperature 20°C was found to be $+0.1447\times10^{-6}$ for oxygen; -1.64×10^{-10} for hydrogen, and -0.81×10^{-10} for helium. The result found for helium is about 25 times less than Tänzler's value, but when substituted in the formula derived by W. Pauli Jr. for the diamagnetic susceptibility of a monatomic gas, it yields a result bearing on the dimensions of the atom which is compatible with our knowledge obtained from other sources.

SEVERAL papers have been published dealing with the magnetic properties of one or more of the gases oxygen, hydrogen and helium, but the results obtained by a considerable number of observers are not in as close agreement as is desirable. It was thought worth while, therefore, when the opportunity[1] presented itself to undertake further research on the magnetic properties of these gases.

The method used in the present investigation consists in balancing the gas magnetically against a liquid of known susceptibility by varying either the pressure of the gas or the common temperature. The device used for this purpose was a manometric balance.

The Manometric Balance

A diagrammatic sketch of the device is shown in Fig. 1. Here, A is a cylindrical metal container with a cover B. Inside the container is a cylindrical glass vessel C, connected to which is a glass tube D, which passes through the bottom of the container and is then bent as shown. The gas inlet E to the container is shown on the left. Opposite the inlet

[1] The appointment of the senior writer as Ernest Kempton Adams fellow enabled him to secure the collaboration of Mr. Hector.

there is a hole through the wall of the container through which passes a glass tube *F* of small bore. After being bent as shown this tube is constricted at *G* and then joined with the tube *D*.

The angular region between the vessel *C* and the container *A* is filled with cement up to the neck of the vessel; and the tube *F* where it passes through the container is firmly cemented to it. The cover *B* of the container is secured in place by screws, and a gasket between cover and container ensures that no gas escape between them. Through the center of the cover there is screwed a hollow insert plug *H* with a contracted opening at the top to admit connection by means of small copper tubing to a manometer. Upon removal of the plug the liquid can be conveniently introduced into the vessel *C*.

After the liquid is introduced and before the manometer opening is closed the gas to be investigated is allowed to flow through the device

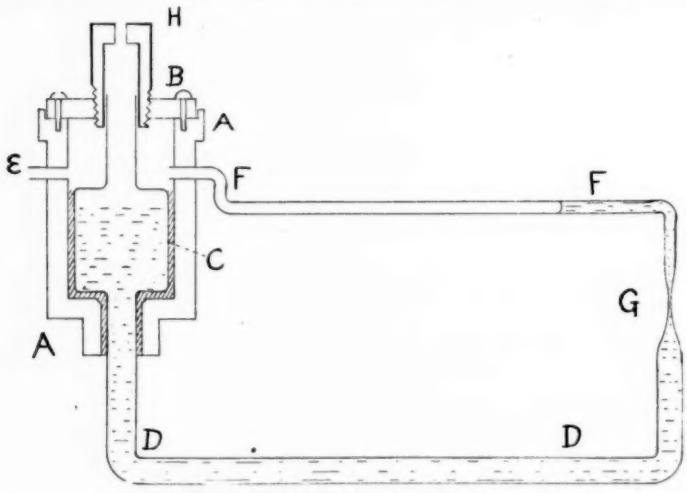


Fig. 1. Manometric balance.

until the air has been thoroughly flushed out. Then the manometer opening is closed and the device arranged so as to bring the meniscus in the tube *F* between the poles of a powerful electromagnet. When the device is properly adjusted the tube *F* is very nearly horizontal, which is the position for maximum sensitivity.

The motion of the meniscus in the tube *F* one way or the other when the magnet is excited enables the observer to decide whether the gas is more magnetic or less magnetic than the liquid. To detect very small motions a microscope is used. This is focussed upon a colloidal particle of gum-mastic suspended in the liquid at the point of contraction *G* of the tube *F*. At this point the movement of the liquid is much greater

than in the unconstricted portion of the tube and the observer may estimate with remarkable nicety the magnetic equality of gas and liquid.

Auxiliary Apparatus. A purifying tube for the gas was introduced between the supply tank and the balance; likewise a temperature regulation coil, this coil being placed in a water bath at room temperature.

A thermostat served to regulate the room temperature.

An ordinary mercury manometer was used to measure the gas pressures when the latter did not exceed two atmospheres. For greater pressures a mercury manometer specially designed for the purpose was used. This was capable of measuring with great accuracy pressures as high as 26 atmospheres.

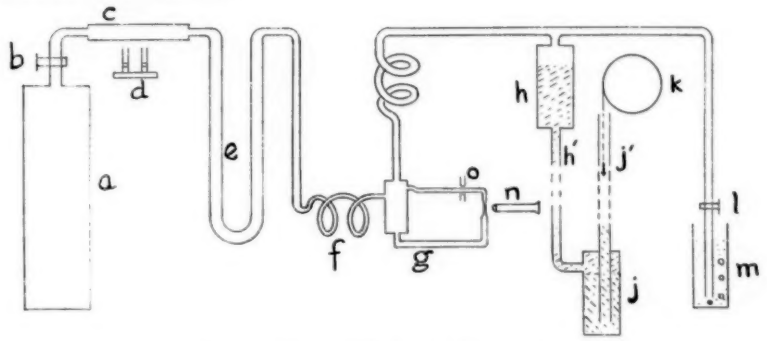


Fig. 2. General diagram of apparatus.

a Gas tank
b Throttle
c Purifying tube
d Bunsen burners
e Drying tube
f Cooling coil
g Balance
h Kerosene reservoir
h' Kerosene tube
j Mercury cistern
j' Mercury tube
k Iron wheel
l Outlet valve
m Water bottle
n Microscope
o Magnet poles

Referring to the general diagram given in Fig. 2, the steel manometer tube *j'* was about 25 meters long with an approximate bore of 0.5 cm. The cylindrical kerosene reservoir *h* was about 30 cm long with an approximate bore of 7 cm; and the mercury reservoir *h* has approximately the same dimensions.

The iron wheel *k* is mounted upon a fixed axle. The face of this wheel is threaded with 20 threads to the inch. One end of a fine flexible silk covered wire is attached to the first thread of the wheel and then wound between the threads around the face of the wheel. The other end of the wire is attached to a metal sounding bob with a fine point. Insulation around the bob prevents it from touching the walls of the manometer tube; but when the bob is lowered until its point comes in contact with

the mercury column it completes an electric circuit containing a bell which announces the contact. The fixed axle supporting the wheel is threaded and likewise the hole in the hub of the wheel through which the axle passes, so that as the wheel turns and advances along the fixed axle the wire which carries the bob is always in line with the axis of the manometer tube. The diameter of the wheel is such that one turn of the wheel causes the bob to rise, or fall, one meter approximately. One edge of the face of the wheel is indexed. A fractional turn of the wheel corresponding to an elevation or depression of the bob of one half millimeter could be estimated with precision.

The pressure of the gas is communicated to the mercury in the reservoir j through the kerosene in the reservoir h and the connecting tube h'.

Theory of the Method

Under the conditions of experiment the gas and liquid in the manometric balance constitute a system in thermal and magnetic equilibrium, assuming the adjustment to equality of the volume susceptibilities of the gas and liquid to have been made.

The liquid used was in all cases an aqueous solution of nickel chloride.

In accordance with Wiederman's law if m and χ denote respectively mass and magnetic susceptiblity per unit mass and if the subscripts l, w, s refer respectively to solution, water and salt:

$$m_l\chi_l = m_w\chi_w + m_s\chi_s. \tag{1}$$

Let R denote the ratio m_w/m_s; then since $m_l = m_w + m_s$ this equation may be written:

$$\chi_l = (R\chi_w + \chi_s)/(1+R); \tag{2}$$

or, if R_0 denote the value of R which makes $\chi_l = 0$:

$$\chi_l = [(R-R_0)/(1+R)]\ \chi_w. \tag{3}$$

The solutions were obtained by mixing water with a mother solution of nickel chloride. For the concentrations used it was found that the density of the solution could be calculated with adequate precision on the assumption of no change of volume of the constituents upon mixing, from the formula:

$$\rho_l = [(1+R)/(1+R-K)]\ \rho_w, \tag{4}$$

where:

$$K = (1-\rho_w/\rho_c)\ m_c/m_s, \tag{5}$$

ρ denoting density and the subscript c referring to the mother solution.

If volume susceptibility be denoted by κ, then $\kappa_l = \rho_l\chi_l$ and from (3) and (4) we obtain:

$$\kappa_l = [\rho_w\ (R-R_0)/(1+R-K)]\ \chi_w. \tag{6}$$

It is possible through elimination of R from this expression with the aid

of (4) to express κ_l as a linear function of ρ_l and of quantities which are constant at a given temperature. It thus appears that $d\kappa_l/d\rho_l$ is constant for an isothermal change in the state of the solution.

Equations for a paramagnetic gas in contact with a solution. Under the conditions of experiment the gas was of necessity always contaminated with water vapor. When a magnetic balance obtains for the gas and solution it is therefore the sum of the volume susceptibilities of the gas and vapor which is equal to the volume susceptibility of the solution. But even if the vapor be supposed saturated the susceptibility of the vapor is negligibly small in comparison with that of a paramagnetic gas at normal pressure or greater. The pressure of the gas, which will be denoted by p will not, however, be the same as the observed pressure, which will be denoted by P. But we may assume that the partial pressure due to the vapor is constant for an isothermal change and write $dp=dP$ for such a change.

For a paramagnetic gas:

$$\kappa = \alpha p, \tag{7}$$

where:

$$\alpha = [\theta_0^2/p_0\theta^2]\ \kappa_0, \tag{8}$$

κ denoting its volume susceptibility at pressure p and temperature θ and κ_0 its volume susceptibility for the standard pressure p_0 and the standard temperature θ_0.

Now, supposing gas and liquid to be in magnetic equilibrium, since for a change at constant temperature $d\kappa_l/d\rho_l$ is constant, as pointed out above, and since from (7) $d\kappa/dp$ is also constant it follows that for an isothermal change in the state of the system:

$$d\rho_l/dp = d\rho_l/dP = A, \tag{9}$$

where A is a constant.

From (6) and (7):

$$\alpha p = [\rho_w(R-R_0)/(1+R-K)]\ \chi_w. \tag{10}$$

Upon differentiation of this expression, assuming an isothermal change, and taking (4) and (9) into account we obtain:

$$\alpha = -[A(1+R_0-K)/K]\ \chi_w. \tag{11}$$

Again assuming an isothermal change and taking P as independent variable we obtain upon differentiation of (4) and taking (9) into account:

$$A = -[\rho_w K/(1+R-K)^2]\ dR/dP,$$

which upon integration gives:

$$K/(1+R-K) = AP/\rho_w + 1/a, \tag{12}$$

where $1/a$ is a constant of integration.

Now when $R=R_0$ we must have $P=0$ and hence:

$$K/(1+R_0-K) = 1/a, \tag{13}$$

a formula from which R_0 may be found after a is known.

From (8) and (11) and (13) we now find the working formula for a paramagnetic gas:

$$\kappa_0 = aA\frac{p_0\theta^2}{\theta_0^2}\chi_w. \tag{14}$$

The constant A is found from two observations at the same temperature of the density ρ_l and the corresponding pressure P in accordance with (9); and the constant a is determined from (12) by an observation of the equilibrium pressure P corresponding to a solution having the known ratio R, the constant K being also known.

Equations for a diamagnetic gas in contact with a solution. In the experimental determination of the susceptibility of a diamagnetic gas the most convenient method was found to be one which utilized the property of the solution of changing its susceptibility with temperature.

The procedure was to vary the common temperature of the gas and solution a small amount $\Delta\theta$, thus disturbing the magnetic equilibrium of the gas and solution, and then to change the pressure by an amount ΔP so that equilibrium was again established.

Neglecting the very small change in the susceptibility of the water vapor contained in the gas and also the small change in the susceptibility of the gas due to the small change of temperature we have:

$$\frac{\partial\kappa}{\partial P}\Delta P = \frac{\partial\kappa_l}{\partial\theta}\Delta\theta. \tag{15}$$

For a diamagnetic gas:

$$\kappa = (p\theta_0/p_0\theta)\ \kappa_0. \tag{16}$$

By differentiation of this expression we obtain:

$$\partial\kappa/\partial p = \partial\kappa/\partial P = (\theta_0/p_0\theta)\ \kappa_0. \tag{17}$$

By differentiation of (6), neglecting the small change in ρ_w and noting that χ_w, R and K are not dependent on the temperature we find:

$$\frac{\partial\kappa_l}{\partial\theta} = -\left[\rho_w\frac{dR_0}{d\theta}/(1+R-K)\right]\chi_w. \tag{18}$$

We now obtain from (15) (17) and (18) the working formula for a diamagnetic gas:

$$\kappa_0 = -\frac{dR_0}{d\theta}\frac{\Delta\theta}{\Delta P}\frac{p_0\theta}{\theta_0}\frac{\rho_w}{1+R-K}\chi_w. \tag{19}$$

The quantity $dR_0/d\theta$ was determined in the course of the experiments on oxygen and $\Delta\theta/\Delta P$ was determined by experiments on the diamagnetic gas in question with the manometric balance.

Oxygen

The gas was prepared electrolytically from a solution of caustic soda. According to the analysis furnished by the company making the gas it

contained 99.6 to 99.7 per cent oxygen with traces of hydrogen, water-vapor and caûstic soda.

Before the gas was introduced into the manometric balance it was passed through a hot paladium sponge, causing the hydrogen to combine with oxygen to form water vapor. Errors due to the presence of water-vapor were avoided as pointed out in the theoretical discussion above.

Chemical analysis of the mother solution from which the solutions used in the experiments were prepared by mixing with water showed it to contain 0.20681 of nickel chloride per gram of solution, with impurities amounting to 0.08, of which cobalt was doubtless the most important. From this result the value of the constant K was calculated to be 0.88102.

After the solution had been introduced into the balance the gas was allowed to flow steadily through the device for several hours to make sure that all air was flushed out.

In the final series of experiments four solutions of different densities were used. Experiments were made on each solution at a temperature closely approximating 296° K in each case, to determine the pressure of the gas when magnetically neutral against the solution. In the following table the results are given, the numbers under P representing pressures expressed in mm of mercury at 0° C.

TABLE I

	Solution 1 $R=29.334$ $\rho l=1.02744$		Solution 2 $R=21.452$ $\rho l=1.03834$		Solution 3 $R=17.004$ $\rho l=1.04893$		Solution 4 $R=12.919$ $\rho l=1.06501$	
	θ	P	θ	P	θ	P	θ	P
	296.13°K	2321	296.20°K	4579	296.20°K	6764	296.10°K	10088
	296.20	2324	296.00	4575	296.10	6764	296.05	10085
	295.84	2322	295.90	4569	295.90	6759	296.05	10082
	295.91	2315	296.00	4573	296.10	6763	296.06	10085
Mean	296.02	2320	296.02	4574	296.07	6762	296.06	10085

From the data here given six values are obtained for the constant A $(=d\rho/dP)$ and also, with the aid of (12) four values for the constant a. The mean values are as follows at 296.04° K:

$$A=(4.838\pm0.001)\times10^{-6}$$
$$a=(53.593\pm0.004)\times10^{-6}$$

Collecting the additional data required in (14) for the calculation of the volume susceptibility of oxygen under the standard pressure p_0 and at the standard temperature θ_0 we have:

$\theta=296.04°$ K, $\quad p_0=760$ mm,

$\theta_0=293.10°$ K, $\quad \chi_w=-0.720\times10^{-6}$.

The value here given for χ_w, the mass susceptibility of water, is that now commonly accepted; it is probably correct to within one half per cent.

Inserting these values in formula (14) and performing the numerical calculations we find:

$$\kappa_0 = 0.1447 \times 10^{-6}$$

for the volume susceptiblity of oxygen under a pressure of one atmosphere at a temperature of 20° C.

Susceptibility determinations on oxygen at different temperatures. The following results were obtained on two solutions:

Temperature	Solution 1 ($R=28.676$)		Solution 2 ($R=12.967$)	
	P	ρ_2	P	ρ_2
285.80°K	2472	1.03002	9729	1.06673
290.10	2464	1.02936	9879	1.06604
293.63	2457	1.02866	9962	1.06532
296.55	2454	1.02798	10048	1.06461

From these data, with the aid of formulas (12) (13) and (14), the results given in Table II were calculated, with the exception of those given in the last column, which were calculated from the value 0.14475×10^{-6} found above for the volume susceptibility of oxygen under a pressure of one atmosphere at the temperature 20° C, it being assumed in this calculation that the susceptibility varies inversely as the square of the absolute temperature in accordance with Langevin's theory for a paramagnetic gas. The symbol κ_1, heading the last two columns denotes volume susceptibility at a pressure of one atmosphere.

TABLE II

Temperature	$\Delta\rho_1/\Delta P \times 10^6$	R_0	$R_0\theta$	$\kappa_1(obs.) \times 10^6$	$\kappa_1(calc.) \times 10^6$
285.80°K	5.0586	48.602	13890	0.1530	0.1522
290.10	4.9467	47.775	13860	0.1472	0.1478
293.63	4.8848	47.320	13890	0.1439	0.1442
296.55	4.8235	46.914	13910	0.1409	0.1414

The rather large differences between corresponding values of κ_1 (obs.) and κ_1 (calc.) are probably to be accounted for by small errors in the determinations of the temperatures of the gas.

The constancy of the numbers under the caption $R_0\theta$ justify, to within the errors of the present series of experiments, the assumption of the validity of Curie's law for paramagnetism in the case of nickel chloride for the temperature range of the present experiments.

HYDROGEN

Hydrogen gas is diamagnetic and, moreover, its susceptibility even under high pressure is very small. In magnetic quality, therefore, it differs little from a vacuum, and it is evident that a liquid which is magnetically neutral against hydrogen will have a value for the quantity R which differs little from R_0. It is difficult, therefore, to make, at a given temperature, a solution of a paramagnetic salt in water which can

be magnetically balanced against hydrogen gas, unless the gas can be subjected to very great pressure. It is easier to make a solution which has somewhere near the susceptibility of the gas and then to bring liquid and gas to magnetic neutrality through the variation of temperature, this procedure being possible since the rates of variation of susceptibility with temperature are very different for the liquid and the gas.

The temperature variations necessary are not great, in fact, in the experiments described below they were always less than 1° C.

The appropriate formula for the calculation of the volume susceptibility of hydrogen under standard pressure and at standard temperature is formula (19). In this formula the quantity R may be replaced by R_0 without introducing appreciable error, since hydrogen differs little from a vacuum in magnetic quality. The values of R_0 and of $dR_0/d\theta$ for the temperatures at which the experiments on hydrogen and helium were performed are found on the assumption that $R_0\theta = \text{constant}$, the value of the constant being taken as 13890 which is the mean of the values under $R_0\theta$ in Table 2. The values of R_0 and of $dR_0/d\theta$ so found are given later in the collected data for hydrogen and for helium.

The experiments on hydrogen have therefore to furnish only the value of the quantity $\Delta\theta/\Delta P$.

To determine this quantity two similar series of experiments were carried out.

An approximately neutral solution of nickel chloride in pure water was made up. This solution could be made slightly magnetic, or diamagnetic, through decrease, or increase, of the temperature by a few degrees. In the series of experiments under consideration the actual procedure was as follows.

The temperature of the room was raised until, by means of the magnetic balance, it was found that the gas under a given pressure was magnetically neutral against the solution; then the pressure of the gas was changed by a considerable amount and the temperature of the room varied until the gas was again neutral against the solution under the new pressure. The increases of pressure and temperature here mentioned are what we have denoted above by ΔP and $\Delta\theta$ respectively. The value of the quotient $\Delta\theta/\Delta P$ required for the calculation of κ_0 by formula (19) was thus determined.

In each of the two series, four observations were made at slightly different temperatures of the pressure and temperature corresponding to magnetic neutrality of gas and solution. The results are given in Table III. The temperature variations were measured by a thermocouple one of whose junctions was in contact with the tube of the magnetic balance

containing the meniscus separating the gas and liquid, the other junction being in melting ice. By calibration against an air thermometer the galvanometer through which the current from the thermocouple passed was found to give a deflection of one scale division for a change in temperature of 0.858°C.

In the first column of the table are found the galvanometer readings corresponding to magnetic neutrality of the gas against the solution; in the second, the corresponding temperatures computed from the data of the first column with the aid of the galvanometer constant; in the third, the readings in pounds of a pressure gauge connected to the supply tank of hydrogen; in the fourth, the actual pressures of the gas expressed in millimeters of mercury, obtained from the data of the third column after calibration of the pressure gauge against a mercury manometer.

TABLE III

Series 1	Gal. defl.	Temperature	Pressure	
1	54.470	303.570°K	28lb	1946 mm
2	.230	.776	80	4600
3	53.880	304.077	145	7980
4	.530	.378	210	11330
Series 2				
5	54.363	304.441	65	3840
6	.098	.668	110	6130
7	53.830	.898	160	8750
8	.570	305.121	210	11330

Mean temperature = 304.36°K

From the data of series 1 six values for $\Delta\theta/\Delta P$ are obtained and weighted, the weight 1 being assigned the values obtained from 1 and 2, 2 and 3, 3 and 4; and the weight 2 to those obtained from 1 and 3, 2 and 4; and the weight 3 to the value obtained from 1 and 4. The same procedure was followed with the observations for series 2.

The weighted mean values so found are:

$$\Delta\theta/\Delta P = 86.2\times10^{-5} \text{ for series 1,}$$
$$\Delta\theta/\Delta P = 90.7\times10^{-5} \text{ for series 2.}$$

Taking the mean of these two results we find:

$$\Delta\theta/\Delta P = 88.5\times10^{-5}$$

The necessary data for the calculation of the volume susceptibility of hydrogen from (19) are collected below:

$p_0 = 760$
$\theta_0 = 293.1$
$\rho_w = 0.99528$
$R_0 = 45.814$
$dR_0/d\theta = 0.1505$
$\theta = 304.36$
$\Delta\theta/\Delta P = 8.85\times10^{-5}$
$\chi_w = 0.720\times10^{-6}$

After inserting these values in the above formula for κ_o it is found that:

$$\kappa_o = -1.64\times10^{-10}$$

for hydrogen under a pressure of one atmosphere at the temperature 20° C.

Soné[2] using an entirely different method arrived at the value -1.659×10^{-10} for the same standard conditions.

Helium

Some preliminary experiments were made on helium, but circumstances compelled the discontinuance for a time of the experiments on this gas before data which could be considered quite satisfactory were obtained. The results given below are therefore to be considered in a measure as tentative.

The helium as furnished contained about 8 per cent of nitrogen. This impurity was removed by passing the gas over hot magnesium which at the same time served to remove any traces of oxygen.

Before the introduction of the helium gas into the measuring apparatus, the latter was flushed out with hydrogen, which was allowed to flow through the apparatus for about four hours to ensure that every trace of air was removed. Then the helium gas was allowed to flow through the apparatus for about one hour before the measurements were begun.

The data obtained for helium in one series of experiments are contained in Table IV which is quite analogous to Table III for hydrogen and the explanation given above for Table III applies equally well to Table IV.

Table IV

	Gal. defl.	Temperature	Pressure	
1	52.030	302.200°K	20lb	1420 mm
2	51.783	.412	115	6380
3	.520	.638	213	11500
		Mean temperature = 302.42°K		

Treating these observations in a manner quite analogous to that used in the case of hydrogen we obtain three values for $\Delta\theta/\Delta P$ from which after weighting and taking the mean value we find:

$$\Delta\theta/\Delta P = 4.35\times 10^{-5},$$

with a mean deviation for the three values of 0.05×10^{-5}.

The data required for the calculation of the volume susceptibility of helium from formula (19) are collected below:

$p_0 = 760$
$\theta_0 = 293.10°\text{K}$
$\rho_w = 0.99600$
$R_0 = 46.110$
$dR_0/d\theta = 0.1524$
$\theta = 302.42°\text{K}$
$\Delta\theta/\Delta P = 4.35\times 10^{-5}$
$\chi_w = 0.720\times 10^{-6}$

Inserting the values here given in formula (19) we obtain:

$$\kappa_0 = -0.807\times 10^{-10}$$

for helium under a pressure of one atmosphere at a temperature of 20°C.

[2] Také Soné, Phil. Mag. **39**, 305, 1920

The susceptibility of helium thus appears to be almost exactly one-half of that of hydrogen under the same standard conditions.

The only other determination for the susceptibility of helium known to the writers is that made by Tänzler.[3] The measurements of Tänzler were relative. Assuming the susceptibility of oxygen to be $+0.123\times10^{-6}$ at 0° C under a pressure of one atmosphere, he found the volume susceptibility of helium under these conditions to be -17.5×10^{-10}. Using the value for oxygen found by us above, the susceptibility of oxygen at 0° C under a pressure of one atmosphere is found to be $+0.167\times10^{-6}$ and on the basis of this value Tänzler's result for helium would be -23.7×10^{-10} at 0° C or -22.1×10^{-10} at 20° C under a pressure of one atmosphere. This result is about 27 times the value found by us for the susceptibility of helium under the same standard conditions. The discrepancy between Tänzler's results and our own is certainly very great.

In this connection a paper by W. Pauli, Jr.[4] is of considerable interest. In accordance with the theory of Pauli for a diamagnetic monatomic gas, if R denote the distance of an electron in an atom from the nucleus and if $\Theta=\Sigma eR^2$, where the summation is over all the electrons in the atom, then:

$$\Theta=1.68\times10^{-20}\ \chi_A,$$

where χ_A denotes the atomic susceptibility obtained from the volume susceptibility at 0° C and 760 mm pressure through multiplication by the mol-volume (22.4×10^3).

From the value of the volume susceptibility for helium found by us above we obtain for the volume susceptibility at 0° C and 760 mm pressure the value 0.866×10^{-10}; hence

$$\chi_A=0.866\times10^{-10}\times22.4\times10^3=1.94\times10^{-6},$$
$$\Theta=1.68\times10^{-20}\ \chi_A=3.26\times10^{-26}.$$

From our knowledge of the order of magnitude of atomic dimensions from other sources, this value for Θ is reasonable. Tänzler's value for the susceptibility of helium leads, on the other hand, to a value of Θ which is far too great.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY,
June 27, 1923.

[3] Tänzler, Ann. der Phys. **24**, 931, 1907
[4] W. Pauli, Jr., Zeit. f. Phys. **2**, 201, 1920

CHARACTERISTICS OF IRON IN HIGH FREQUENCY ROTATING MAGNETIC FIELDS

By Lewi Tonks

Abstract

Characteristics of electrolytic iron in rotating magnetic fields of high frequency, 52,000 to 172,000 and of strength 0.7 to 5 gauss, have been determined by measurements on a disk, 10^{-3} cm thick shellacked to a half inch cover glass. Equations are derived by which H and I can be computed from measurements of the impressed field, the fluxes through a coil with and without the disk in it, and the torque exerted on the disk by the rotating field. This field is produced by means of two Helmholtz coils with axes at right angles, each in series with a separate oscillating circuit, coupled to the main oscillating circuit. The results agree with previous work and show that (1) the susceptibility decreases with increasing frequency; (2) the hysteresis angle tends to increase with frequency and for a given frequency reaches a maximum for I at about 500; (3) the hysteresis energy loss per cycle E_h increases with frequency for constant magnetic intensity and decreases for constant field, the exponent α in the equation $E_h = AH^\alpha$ increasing from 2.9 to 3.37 as frequency is increased from 52 to 172 kilocycles.

THE magnetic properties of iron for static fields, for alternating fields from zero to very high frequencies,[1] and for slowly rotating fields[2,3] are well known, but the behavior of iron in a high frequency rotating field has not, so far, been investigated. Such an investigation, carried out at frequencies from 50,000 to 170,000 cycles per second is described below.

A. Mathematical Theory.

1. *Rotating magnetic field in an iron disk.* As a thin circular disk of iron was used in the experiments, it was necessary to investigate the relations of the magnetic vectors in the disk to the impressed field. Because this problem is of extreme difficulty when the vectors are not constant throughout the disk, it was necessary to be assured of the essential uniformity of the field and polarization under the conditions of the experiment. For this purpose the disk was considered as a very oblate spheroid. When an oblate spheroid of iron is placed in a uniform magnetic field parallel to its equatorial plane, it becomes uniformly polarized

[1] Arkadiew, Phys. Zeits., **14**, 561, 1913
[2] R. Gans, Arch. f. Elektrotechnik **3**, 139, 1915
[3] G. Vallauri, Electrician, **65**, 603, 1910

parallel to the field. If now, the ellipsoid is rotated slowly about its minor axis, the polarization is no longer parallel to the field, but lags behind it, remaining uniform, however, if the field in the spheroid is uniform. Such a uniform polarization gives rise to a uniform demagnetizing field, which, together with the uniform impressed field, gives the required uniform magnetic field throughout the iron. When, instead of rotating the ellipsoid, the magnetic field is slowly rotated in the opposite direction, the consequences are identical.

But when the angular velocity of the field becomes great, two further considerations enter. First, any inertia or viscosity effect in the magnetic particles of the iron would lead to a greater but still uniform lag of polarization behind field, which would vary with the velocity. Second, Foucault currents are generated and give rise to non-uniform magnetic fields in the iron. Under certain circumstances this effect can be handled mathematically, and one of the possible cases which closely approximates that of the disk is that of an infinite thin sheet of iron subjected to a magnetic field rotating in its plane.

2. *Magnetic vectors in an infinite sheet.*[4] For the infinite sheet we have the electromagnetic field equations[5]

$$c \operatorname{curl} \mathbf{E} = -\dot{\mathbf{B}},$$

$$c \operatorname{curl} \mathbf{H} = 4\pi \mathbf{i},$$

and

$$\mathbf{i} = \mathbf{E}/\rho,$$

all in Gaussian units.

Also

$$\nabla \cdot \mathbf{H} = 0,$$

since $\mathbf{H}$ and $\mathbf{I}$ are parallel to the surface and constant at any depth. These equations give

$$\nabla^2 \mathbf{H} = 4\pi \dot{\mathbf{B}}/\rho. \tag{1}$$

When the impressed field rotates, the field $\mathbf{H}$ at any depth is

$$\mathbf{H} = H[\cos(\omega t - \alpha) i + \sin(\omega t - \alpha) j] \tag{2}$$

where i and j are unit orthogonal vectors fixed in the plane of the disk, ω is the angular velocity of the impressed field, α is a lag angle varying with depth, and H depends upon depth also. The intensity of magnetization I, lagging behind H by an angle β, is given by

$$\mathbf{I} = kH[\cos(\omega t - \alpha - \beta) i + \sin(\omega t - \alpha - \beta) j] \tag{3}$$

and it is assumed that the susceptibility k and the angle β are constant for the small variation of H through the sheet. Acccordingly

[4] M. Latour, Inst. Rad. Eng. Proc. **7**, 61, 1919, solves a problem in alternating fields the mathematics of which is directly applicable here. His results are not, however, complete for the present purpose.

[5] In what follows, vector quantities will be denoted by Clarendon type except in the case of Φ for which heavy type is not available. The vector notation of Gibbs is used.

$$k = I/H = \bar{I}/\bar{H}, \tag{4}$$

dashes always denoting average values.

Eqs. (2) and (3), together with $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}$ enable us to write Eq. (1) in terms of H and a only. The solution of the resulting second order partial differential equation subject to the condition that at both surfaces of the sheet

$$\mathbf{H} = \mathbf{H}_s = H_s[\cos \omega t\, i + \sin \omega t\, j]$$

(where $\mathbf{H}_s$ is the magnetic field strength at the surface of the disk) is somewhat long but presents no difficulties, so it will not be reproduced here. On averaging the expression for $\mathbf{H}$ throughout the sheet it is found that

$$\bar{H} = H_s \sqrt{\frac{4(\cosh X - \cos Y)}{(X^2+Y^2)(\cosh X + \cos Y)}} \tag{5}$$

and

$$\sin \bar{a} = \frac{\sqrt{2}\,(Y \sinh X - X \sin Y)}{\sqrt{(X^2+Y^2)(\cosh 2X - \cos 2Y}} \tag{6}$$

where

$$X = \tau \sqrt{\frac{4\pi\omega}{\rho c^2}} \sqrt{\frac{\sqrt{\mu'^2+\mu''^2}+\mu''}{2}},$$

$$Y = \tau \sqrt{\frac{4\pi\omega}{\rho c^2}} \sqrt{\frac{\sqrt{\mu'^2+\mu''^2}-\mu''}{2}},$$

and

τ = thickness of disk,

$\mu' = 1 + 4\pi k \cos\beta$

$\mu'' = 4\pi k \sin\beta$

The qualitative relationships between $\bar{\mathbf{H}}$, $\mathbf{H}_s$, and a are shown in Fig. 1.

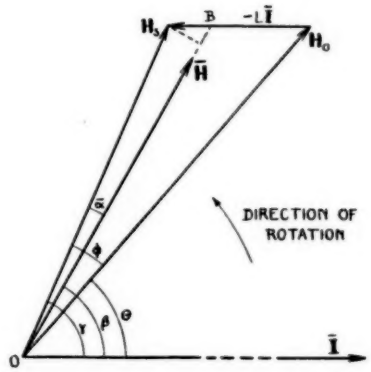


Fig. 1. Magnetic vectors in the disk.

3. *The theory for an infinite sheet applied to the thin disk.* In the sheet the magnetic vectors vary from their surface values with depth, but at a fixed depth there is uniformity over the whole plane. In the disk the vectors vary with depth and surface position too on account of the

variation in penetration. It seems fairly evident, however, that on account of the finite size of the disk, penetration in it at any point will be greater than in the sheet. Hence, although the non-uniformity in the disk is of direction as well as magnitude, the investigation of the sheet should at least set an outside limit for eddy-current effects, and should also give very nearly the vectorial relations in such very flat disks as were used in this experiment. The mathematical results for the infinite sheet were, therefore, applied directly to the disk, due account being taken of the demagnetizing field in the disk. Thus, for the disk, we have,

$$\mathbf{H}_s = \mathbf{H}_0 - L\mathbf{I},$$

where $\mathbf{H}_0$ denotes the impressed field and L the demagnetizing factor of the disk considered as of an oblate spheroid.

Reference to Fig. 1 shows immediately that this vector relation implies the two scalar equations

$$\varphi = \tan^{-1}\left[\frac{L\bar{I}\sin\theta}{H_0 - L\bar{I}\cos\theta}\right] \tag{7}$$

and

$$\bar{I}/H_s = \sin\varphi/L\sin\theta, \tag{8}$$

θ and φ being the angles shown. Another relation to be found from the figure is

$$\beta = \theta + \varphi - \bar{\alpha}. \tag{9}$$

The five equations (5), (6), (7), (8), (9) contain the seven unknown quantities $\bar{I}, \bar{H}, H_s, \bar{\alpha}, \beta, \theta, \varphi$. Therefore, two additional relationships must be supplied by experiment.

4. *The two experimental relationships.* (a) If a coil of area a be wound closely around the disk parallel to a diameter, the e.m.f. induced in the coil by the rotating magnetic field will be a measure of the total flux through one turn of the coil. This flux will be given by

$$\Phi_F = \mathbf{H}_0 a + (4\pi s - aL)\bar{\mathbf{I}},$$

s being the area of a cross section of the disk. In this expression the field of the eddy currents has been neglected in comparison with the polarization flux $4\pi I$, and the back field due to the polarization in the disk has been assumed uniform over the very small area between the disk and the coil. Eq. (5) may be written

$$\Phi_F = \bar{\Phi}_0 + J\bar{\mathbf{I}}$$

where $\Phi_0 = \mathbf{H}_0 a$ is the flux measured by the e.m.f. induced in the coil with the disk removed, and $J = 4\pi s - aL$. Solving this vector equation for the magnitude of $\bar{\mathbf{I}}$ we find that

$$\bar{I} = (1/J)(\Phi_F - \Phi_0\cos\theta - \Phi_0^2\sin^2\theta/2\Phi_F + \ldots .) \tag{10}$$

(b) If the disk, removed from the coil, be now suspended by a torsion fiber so as to be capable of rotation about an axis through its center

perpendicular to its plane, it will experience a torque, R, due to the rotating field, of magnitude

$$R = VIH_0 \sin\theta \tag{11}$$

where V is the volume of the disk, the torque due to the action of the field upon the eddy-currents being neglected, since this was less than $1/4\pi k$ which is equal to 0.4 per cent of the other at a maximum.

5. *Solution of the equations.* An approximate value of I, Eq. (10), given by

$$I = (\Phi_F - \Phi_0)/J \tag{11'}$$

is substituted in Eq. (11) which gives an approximate value for θ, which, when substituted in Eq. (10), yields the final value of I. This, when put in Eq. (11), gives the final value of θ; θ and I together, used in Eq. (7), give φ; and this with θ put in Eq. (8) gives the ratio I/H_s.

It follows, then, from Eqs. (4) and (5) that

$$k = \left(\frac{I}{H_s}\right)\sqrt{\frac{(X^2+Y^2)(\cosh X+\cos Y)}{4(\cosh X-\cos Y}}. \tag{12}$$

Equations (6), (9), and (12) remain for the determination of k, β, and, incidentally $\bar{\alpha}$. For this, Eqs. (6) and (12) were expanded into convergent series:

$$\sin\bar{\alpha} = (1/6)kT\cos\beta[1-(7/30)kT\sin\beta+\ldots.], \tag{12'}$$

$$k = (\bar{I}/H_s)[1+(1/6)kT\sin\beta-(1/180)k^2T^2\sin^2\beta+(1/50)k^2T^2\cos^2\beta+\ldots] \tag{12''}$$

where $T = 8\pi^2\omega\tau^2/\rho c^2$.

An approximate value for $\bar{\alpha}$ was found by using $k = (\bar{I}/H_s)$ and $\beta = \theta+\varphi$ in the coefficient of the right hand member of Eq. (12'). This value of $\bar{\alpha}$ was used to give a first approximation to β by Eq. (9). With this value of β, k was obtained by Eq. (12''). A new value of $\bar{\alpha}$ computed by Eq. (12') and substituted in (9) gave a check on the adopted value of β and on the computation of k.

Thus it becomes possible to make a complete study of the magnetic vectors **H** and **I** in the disk at any desired frequency through observation of the four quantities H_0, Φ_F, Φ_0, and R.

6. *Energy losses in the disk.* The hysteresis loss in the disk for one cycle is, per unit volume,

$$E_h = \frac{1}{4\pi}\int_{\text{cycle}} \mathbf{H}\cdot d\mathbf{B},$$

which reduces to

$$E_h = \int \mathbf{H}\cdot d\mathbf{I} = 2\pi\bar{H}\bar{I}\sin\beta \tag{13}$$

in the present case.

The eddy current dissipation does not appear explicitly in the calculations and it is interesting to know approximately what ratio it bears

to the hysteresis loss. For the power dissipated

$$\text{Total} = \text{Eddy current} + \text{Hysteresis}.$$

Referring to Fig. 1:

$$\text{Total} = \omega H_0 \bar{I} \sin\theta,$$
$$\text{Hysteresis} = \omega \bar{H} \bar{I} \sin\beta.$$

Therefore

$$\frac{\text{Eddy current}}{\text{Hysteresis}} = \frac{H_0 \sin\theta - \bar{H}\sin\beta}{\bar{H}\sin\beta}.$$

From the diagram also

$$H_0 \sin\theta = OB \sin\beta,$$

and

$$OB = \bar{H} + (H_s - \bar{H}) + \bar{H} \sin\bar{\alpha} \cot\beta.$$

Making these substitutions in the expression for the energy ratio, we have

$$\frac{\text{Eddy current}}{\text{Hysteresis}} = (H_s/\bar{H} - 1) + \sin\bar{\alpha}\cot\beta,$$

and by (4)

$$\text{Energy ratio} = (kH_s/\bar{I} - 1) + \sin\bar{\alpha}\cot\beta.$$

Then by (12′) and (12″)

$$\text{Energy ratio} = kT/6\sin\beta. \tag{14}$$

B. Apparatus and Adjustment

7. The iron used in these tests was specially rolled electrolytic iron of thicknesses 5, 10, and 17×10^{-4} cm, determined by weighing. The disks were annealed in hydrogen by heating to 1000° C and cooling slowly. Then they were mounted with melted shellac on one-half inch circular microscope cover glasses, and the outside surface of the iron was covered with a thin coating of liquid shellac. Finally each disk was carefully demagnetized.

8. The horizontal component of the earth's magnetic field was neutralized with a Helmholtz coil[6] 30.5 cm in diameter.

9. The suspension system and mounting is shown in Fig. 2. The iron disk was stuck to the bottom of the amber carriage with vaseline, and this was suspended just clear of the bottom of the glass tube. This tube projected into the Helmholtz coils (see §10) so as to bring the disk slightly above the center of the rotating field and very close to the test coil (see §11).

10. Two small concentric Helmholtz coils, *H.C.* I of 100 turns and 5.95 cm diameter and *H.C.* II also of 100 turns but 5.07 cm in diameter (Fig. 2), were used to give the rotating field and assure its instantaneous space uniformity. The axes of these double coils were necessarily at

[6] Double coil of the kind used in the Helmholtz type of tangent galvanometer.

right angles to each other. The axes were also made horizontal so that the magnetic field should be parallel to the disk, but this adjustment was not critical.

11. A test coil (Fig. 2) of 18 turns mounted on a rotating spindle with divided circle stood just below the center of the Helmholtz coils and

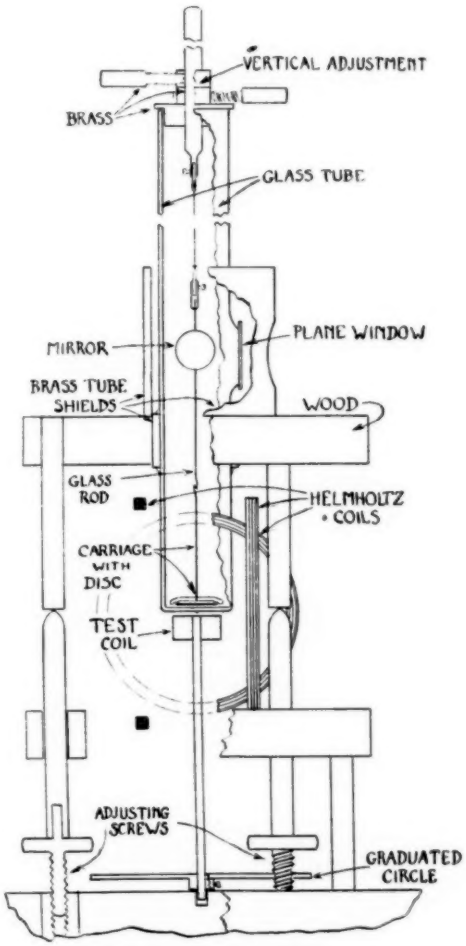


Fig. 2. Suspension system.

immediately below the glass tube enclosing the suspension system. It was used in adjusting and measuring the rotating field.

12. The electrical system (Fig. 3) for obtaining the rotating field consisted of three units, the main oscillating circuit which was driven by vacuum tubes and two auxiliary circuits coupled to it. The two auxiliary circuits were similar to each other, each containing one of the Helm-

holtz coils (see §10), an inductance coupled variably to the main circuit, and a variable condenser.

Tuning was accomplished by opening circuits I and II and adjusting the main circuit to the desired frequency with a wave-meter. Coupling M_1 was made very loose, and circuit I was closed and tuned for resonance by means of the test coil and detection circuit (see §13). Circuit I was then detuned by increasing C_1 until the magnetic field as determined by the test coil was 0.7 its resonance value. At this setting the currents

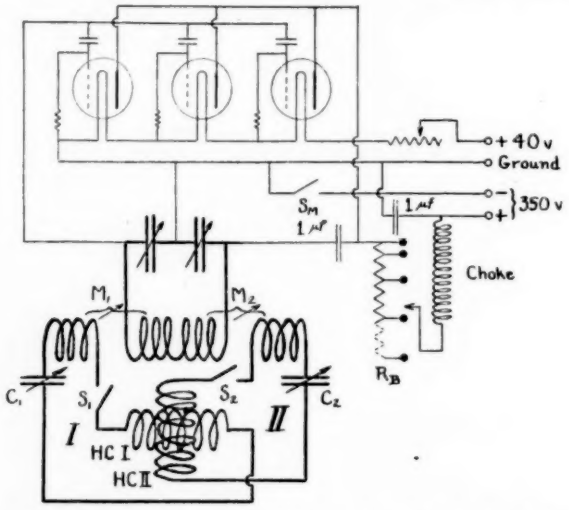


Fig. 3. Electrical circuit for obtaining the rotating field.

in the main circuit and circuit I were 45° out of phase. Opening circuit I, the same procedure was applied to circuit II, save that in detuning from resonance C_2 was decreased. Then the currents in the main circuit and circuit II were 45° out of phase, and when circuit I was closed the phase difference in the two auxiliary circuits was 90°—a condition necessary to obtain a rotating field. But it was also necessary that the magnitudes of the two component fields be equal. This was accomplished by using some data previously obtained, which gave the values of M_1 and M_2 giving equal fields in the Helmholtz coils when I and II were in resonance, and hence when they were similarly detuned.

This whole adjustment was, however, only rough, and the final tuning was done with the test coil. (1) By turning it so as to be parallel first to *H.C.* I (0° position) and then to *H.C.* II (90° position), the magnitudes of the two components were tested. Adjustment for equality was made by changing the frequency of the main circuit slightly. (2) By turning the test coil so as to be in the 45° position and then rotating it to −45°,

the relative phase was tested and adjustment for equality of e.m.f. in the two positions was made with a vernier condenser in multiple with C_2. These adjustments were not quite independent so that they had to to be made by successive approximation.

In making a set of readings the magnitude of the rotating field had to be changed. This was accomplished with a specially designed eleven-point rheostat R_B in the plate current supply.

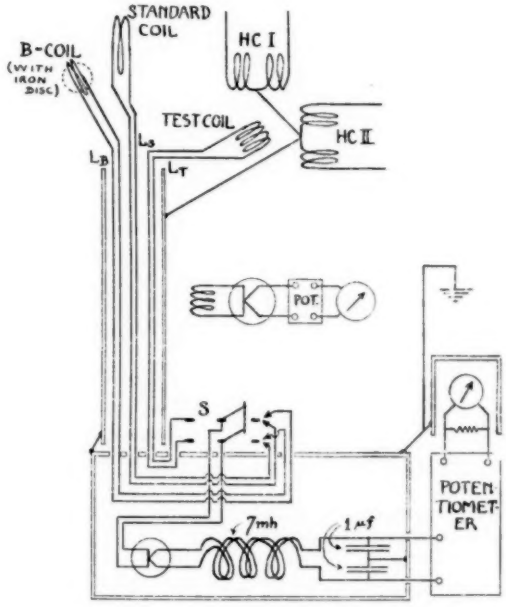


Fig. 4. Electrical measuring system.

13. The electrical measuring system (Fig. 4) consisted of an 880-ohm Western Electric vacuum thermocouple, and a potentiometer reading directly to 5×10^{-7} volts. The essentials of the measuring circuit are shown in the simple circuit in the center of Fig. 4. The D-P, D-T switch of the complete circuit was used to measure successively the e.m.fs. induced in various coils placed in the rotating field. The grounding (not shown in Fig. 3), shielding, choke coil, and condensers were necessary to eliminate electrostatic and other stray induction effects.

14. A "standard" coil and three "B-coils" were others used for measurements in the rotating field. The standard coil had a single layer of 16 turns 1.64 cm in diameter. In order to eliminate extraneous induction, the standard coil leads were laid close to each other, but to avoid the attenuation that would arise from distributed capacity, the leads were not twisted. This permitted the e.m.f. induced in the coil to be

measured directly from the direct-current calibration of the thermocouple, which was checked from time to time, and the known area of the standard coil.

Two of the B-coils, B-38 and B-91, were the first used for the induction measurements. B-38 had 38 turns of No. 36 enameled copper wire wound in four layers of 11, 10, 9, and 8 turns respectively. B-91 had 91 turns of No. 40 enameled copper wire wound in 13 layers of 13, 12, . . ., 1 turn respectively. The openings of the coils were a snug fit for a 0.5 inch circular microscope cover glass. It is evident that the distributed capacity of coils such as these might affect measurements with them profoundly, especially as the leads were twisted to prevent extreneous induction. The elimination of this source of error is explained below.

The third B-coil, B-11, was made later to check the measurements with the others. It had 11 turns of No. 36 enameled copper wire wound in two layers of 6 and 5 turns, respectively.

C. Measurement of Φ_F and Φ_0

15. *Standardization of coils.* As explained above, the full e.m.f. induced in the single-layer standard coil could be expected to appear across the thermocouple heater, and this expectation was borne out by the smallness of the effect of frequency on the readings with the other multilayer coils. While, therefore, these other coils acted as if they had smaller areas at the higher frequencies, the "apparent" area of the standard was constant and equal to its true area. Now, at any frequency, in the rotating field H_0

$$E_s = (10^{-8}/\sqrt{2})\omega A_s H_0 \tag{15A}$$

and

$$E = (10^{-8}/\sqrt{2})\omega A_{\omega,T} H_0 \tag{15B}$$

$$V_0 = (10^{-8}/\sqrt{2})\omega A_{\omega,B} H_0 \tag{15C}$$

where

E_s = e.m.f. in standard coil as measured;
E = e.m.f. in test coil as measured;
V_0 = e.m.f. in B-coil without iron disk, as measured;
ω = 2π times the frequency;
A = complete area of a coil.[7] Subscripts 0 and ω refer to true and apparent (at frequency $\omega/2\pi$) areas, respectively, and subscripts S, T and B refer to standard, test, and B-coil, respectively,

These are practically equations of definition for $A_{\omega,T}$ and $A_{\omega,B}$. Combining (15B) with (15A) and 15(C) in turn:

$$A_{\omega,T} = (E/E_s)A_s \tag{16A}$$

$$A_{\omega,B} = (V_0/E)A_{\omega,T} \tag{16B}$$

[7] Complete area = Σ area of turn, for all turns in coil.

E and E_s were measured by placing the test coil and standard alternately in the same field, and A_s being known, the apparent area $A_{\omega,T}$ was calculable. Similarly, $A_{\omega,B}$ was found by using (16B).

It was also essential to know a_B, the average true area per turn of a B-coil. For this purpose a B-coil and standard coil were connected in series to a galvanometer. The B-coil was placed parallel to and in the center of *H.C.* II and the deflection δ_B of the galvanometer noted when a known current i_B through the Helmholtz coil was reversed. Then the standard coil was put in place of the B-coil and the current adjusted until δ_s, the deflection with it, was very nearly equal to δ_B. This precaution was taken as the galvanometer was not ballistic. The true total area of the B-coil was then given by

$$A_{O,B} = (\delta_B i_s / \delta_s i_B) A_s$$

and the average area per turn of the B-coil was $a_B = A_{O,B}/N_B$ where N_B is the number of turns in the B-coil. The true area, $A_{O,B}$ was determined for both B-38 and B-91; the apparent area $A_{\omega,B}$ was determined for each coil at each frequency at which the coil was used, and $A_{\omega,T}$ was determined at all frequencies.

The true area of B-11 was not measured and it was assumed that the apparent area at all frequencies was the same as the true area. There would have been some difficulty in measuring the area ballistically, and as the difference in areas in the case of B-11 would be still smaller than the small differences in the cases of B-38 and B-91, the measurement was deemed unnecessary.

16. *Measurement of total flux.* The disk to be measured was placed in one of the B-coils; and this B-coil was connected to the leads L_B, Fig. 4, which go to the switch S. Then, with R_B set for maximum magnetic field, and S to the left, the rotating field was adjusted for isotropy as explained in §12. With the test coil at 90° the potentiometer was read. The B-coil with the iron disk was immediately set in the rotating field parallel to and immediately above the test coil, S was thrown to the right and the potentiometer reading P_F was noted. P_F is a measure of Φ_F [§4(*a*)]. As a check the B-coil was removed and further readings were taken on the test coil, one in the 90° position which had to duplicate the first to 1 per cent (this meant 0.5 per cent in e.m.f. as the thermocouple follows a square law), and one in the 0° position to test the isotropy of the field. The one reading of the wave-meter for the whole run was then made. Then the rotating field was decreased in strength, was adjusted again, and another reading made.

The two 90° potentiometer readings on the test coil were averaged giving P_E, which is a measure of Φ_0 [§4(*a*)]. By the thermocouple cali-

bration E was found from P_E, and V_F (the e.m.f. in the B-coil with the iron disk) was found from P_F. V_0 was calculated from E by means of the coil constants in Eq. (16B), and H_0 was calculated from E by Eq. (15B). Then the fluxes Φ_0 and Φ_F were found from V_0 and V_F, respectively, by the formula

$$\Phi = (\sqrt{2}\ 10^8/\omega N_B)(A_{O,B}/A_{\omega,B})V.$$

17. *Possible B-coil errors.* There are two factors affecting measurements with the B-coils which have not yet been mentioned. One is that the current in the thermocouple heater circuit, small though it is, may, in traversing the B-coil, give rise to a magnetic field which is not negligible compared to the impressed field. This effect would be approximately proportional to frequency and very roughly proportional to the square of the number of turns in the B-coil. If this cause were operative, then at a given frequency the B-coil with the greater number of turns would give the lower intensities of magnetization. For increasing frequency a given B-coil would show decreasing intensities, and of two B-coils the one with the greater number of turns would show the greater decrease. Within the limits of accuracy these phenomena were noted in the case of B-91 and B-38.

The second factor affecting the B-coil measurements is that the B-coil with iron may introduce far more impedance into the detection circuit than does the B-coil without iron, and the reactance thus introduced might be comparable with the resistance of the thermocouple heater. In this case again there would be an apparent decrease in intensity of magnetization with increasing frequency and increasing turns on the B-coil. A rough idea of the possible magnitude of this effect was obtained by measuring the inductance of the B-coils with disk X_3(0.00100 cm thick) in them. The results showed that B-11 was the only coil which could be relied on absolutely and without further evidence to give readings unaffected by this second factor.

But the agreement between measurements with B-38 and B-11 at all frequencies used gives the necessary additional testimony that under the conditions of the experiment B-38 is free of excessive reactance, and this agreement shows further that the first effect also is absent from both coils. Hence the B-11 and B-38 measurements were regarded as reliable and were given equal weight where it was possible to make measurements with both coils, otherwise the B-38 measurements alone were used.

D. Measurement of the Torque R on the Disk

18. The suspension system was set up with the coarsest suspension, the disk was placed on the carriage, and the whole placed in the glass

tube support. The carriage was lowered until it came just clear of the bottom of the tube and the tube was leveled until the carriage hung free. The head was then rotated until a light spot, reflected from a mirror on the system, came near zero on a scale 110 cm distant.

At this point the rotating field was started by closing switch S_M (Fig. 3) and the suspension system began swinging violently. However, by the time that the rotating field had been adjusted and the potentiometer set for a balance with the thermocouple connected to the test coil, the oscillation had decreased to 2 or 3 mm in a total deflection of 20 cm, and the reading was estimated. The rotating field was immediately stopped by opening S_M and the zero of the suspension system was taken from the swings. P_E, the potentiometer setting, was then noted, and E found from the couple calibration. The torque R was immediately calculated from the deflection, but on account of the closeness of the disk to the test coil the value of E gave [Eq. (15B)] the value of the impressed field decreased by a small amount (5 to 10 per cent of the impressed field) proportional to the polarization of the disk.

Accordingly, the measurement detailed above was followed by one to determine the diminution. The field was adjusted as before and the potentiometer read. Then the whole suspension system, disk and all, was removed, and 90° and 0° readings on the potentiometer were taken. Finally the system was replaced and 0° and 90° readings were again taken as check. A mean of the first, fourth, and fifth readings led to an apparent value of H_0, and the second and third readings gave the true H_0. The difference between the two, divided by the approximate value of I computed from Eq. (11′) gave the factor of proportionality to be used in correcting the field measured by the test coil. Each time that the suspension fibre was changed in the course of the measurements the distance between disk and test coil was changed, and this factor had to be redetermined.

E. Results

19. The most careful and complete measurements were made on disk X_3 (0.00100 cm thick), because it was found that the intensity measurements with the other two disks, .0005 and .0017 cm thick varied with the orientation of the disk in the B-coil. This apparent anisotropy arose, no doubt, through some non-homogeneity of the disk, probably variable thickness of the iron. This meant that errors of unknown amount were introduced into readings on these two disks, so results with them were regarded only as general checks on the determinations with X_3.

The final results on X_3 are given graphically in Figs. 5 to 10. By way of explanation it may be said that in Figs. 5 through 8 the solid line portion of the curve is that portion which is confirmed by measurements with B-11, the broken line indicates that the portion is calculated from B-38 measurements, and the dotted line shows the B-91 measurements. Also, the curves marked G are determinations made by Gans[2] working with a slowly rotated disk of annealed Remy steel, and the curve marked V is based on measurements of Vallauri.[3]

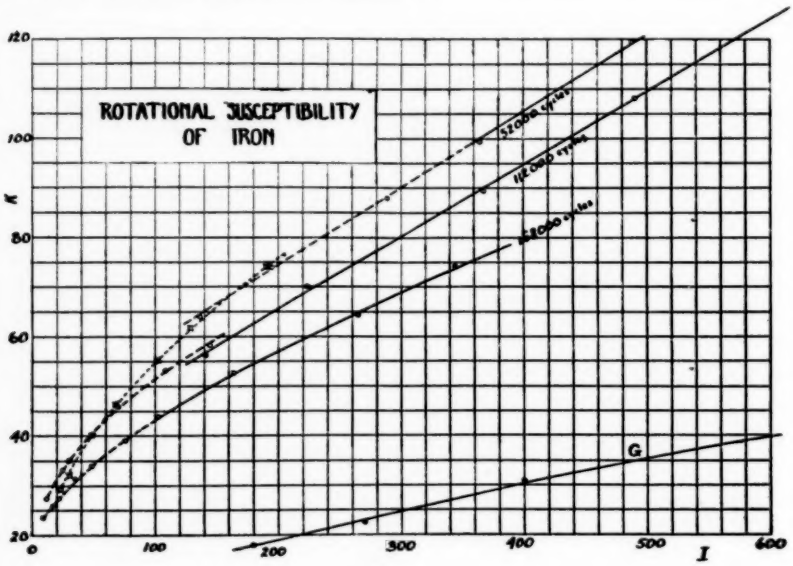


Fig. 5

The susceptibility against intensity of magnetization curves of Fig. 5 have the same general shape as the curve for low frequency alternating fields. Their distinctive feature lies in the decrease of k with increase of frequency. The 52000 cycle results present an apparent contradiction at the lower intensities,but the anomalous portion of the curve was only included in the published results after a great deal of debate concerning the validity of these measurements with B-91, which led to the conclusion that a complete ignoring of these (starred) points was not quite warranted, so they have been included subject to doubt. The 112,000 cycle curve was continued in the way shown because there is a point on the curve at 686,137. These results are in qualitative agreement with measurements in stronger undamped alternating fields over a wider range of frequencies which have been made by Schames.[8] At H (effec-

[8] Schames, Ann. der Phys. **27**, 64, 1908

tive) = 21 he found a rapid decrease in k from 60 at 2400 cycles to 37.4 at 23,500 cycles, and then a linear decrease to 29.4 at 152,000 cycles.

The angle of lag against intensity of magnetization curves Fig. 6 show no behavior as simple as that of the intensity of magnetization curves. Good quantitative agreement with Gans' results is not to be expected

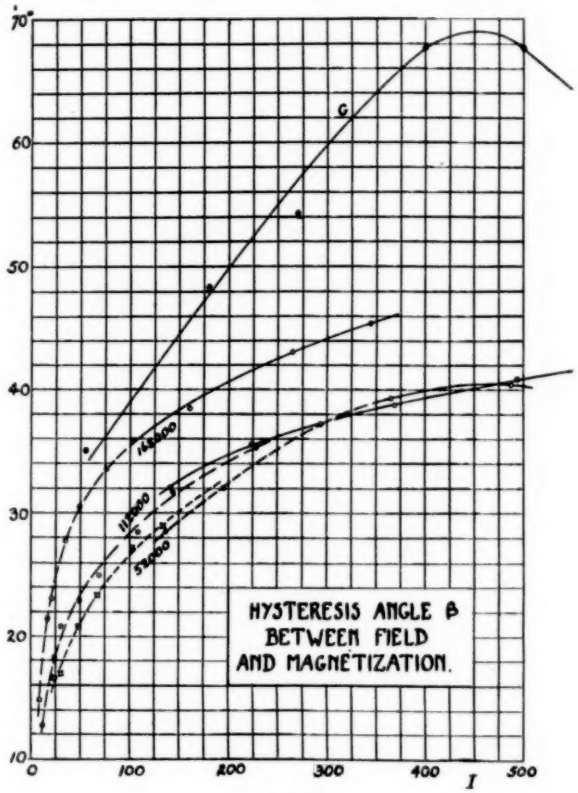


Fig. 6

because of the different iron used, but the general shape of the curves should be the same. Within the range of the present experiments the chief feature of Gans' curve is a maximum at $I = 450$. The new curves are similar, for the 52,000 cycle curve indicates a maximum at about 500, and the 112,000 cycle curve indicates one at about 600 when the point 686, 41.3° is taken into account.

Very low fields could not be measured in the present investigation, but the behavior of β at very small fields would be of considerable interest, for in familiar theories of magnetization the angle β is a measure of the number n of crystal aggregates which have traversed a state of instability during the application of the field. If, for example, β were

found to be constant, it would probably mean that n was proportional to H, and if β were proportional to H, then probably n would have to vary as H^2. Any theory of ferromagnetism would have to conform to such a conclusion.

In Fig. 7 the 0.546 power of the hysteresis per cycle per cm³ is plotted against I because the transformed curve can be shown without splitting into two or more branches. For convenience, the E_h as well as the

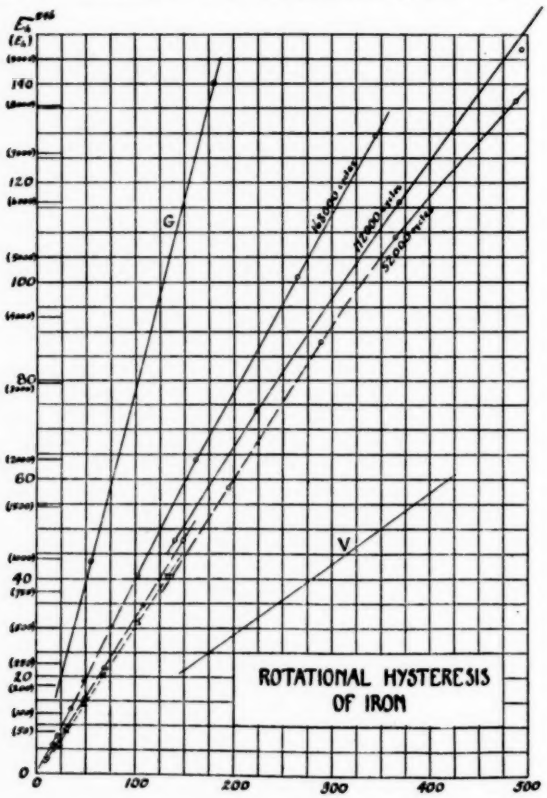
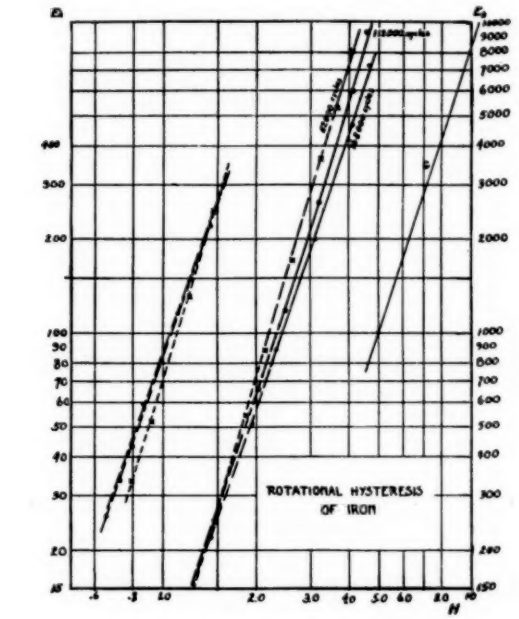


Fig. 7. Rotational hysteresis $E_h^{0.546}$ as function of I.

$E_h^{0.546}$ scale is marked. Fig. 8 also gives E_h, but now plotted against the magnetic field strength H. The straight line logarithmic plot shows that $E_h = AH^{2.90 \text{ to } 3.37}$ in the region covered. Comparison of the two figures, 7 and 8 shows that for constant I, hysteresis increases with increasing frequency, whereas for constant H, hysteresis decreases (save for the doubtful starred points). Had it been possible to experiment with higher fields a maximum of hysteresis loss would, no doubt, have been found at $I = 1200$ to 1300, with the maximum value of hysteresis somewhere between 10,000 (Gans[2]) and 100,000 (Vallauri[3]) ergs per cm³.



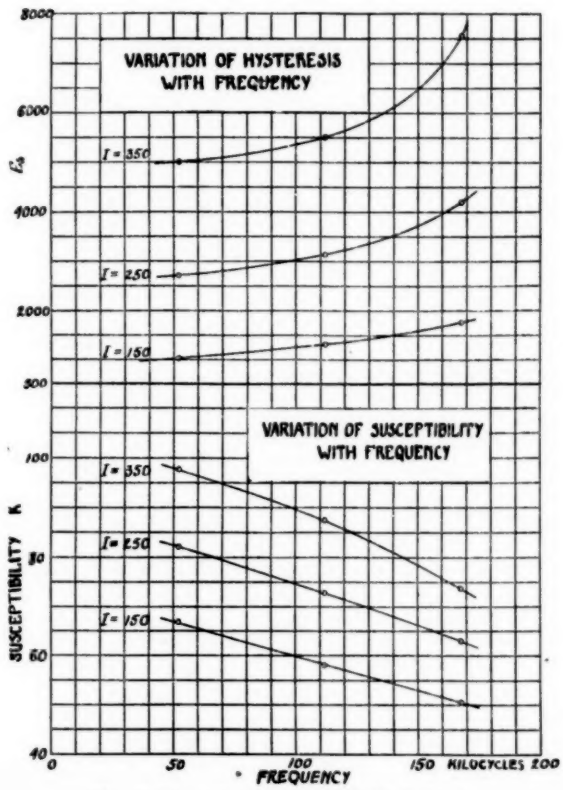


Fig. 8 (above). Figs. 9 and 10 (below)

The points in Figs. 9 and 10, which are self-explanatory, were taken from the curves Figs. 5 and 7.

The table gives some values of the energy ratio expressed by Eq. (14) and also shows the completeness of penetration, $\bar{H}/H_S$ when penetration was least complete, namely, at the maximum fields used.

		Disk X_3 (0.00100 cm thick)				
Frequency	Energy Ratio*	k	H	$\bar{H}/H_s$	β	$\bar{a}$
168,000	.146	74.3	4.70	.93	45°00	.°45′
112.000	.191	137.	5.01	.93	41°20′	4°55′
52,000	.078	119.4	4.09	.97	40°26′	2°11′

* Eddy current to hysteresis

In conclusion I wish to thank Professor J. H. Morecroft for his ever-willing advice concerning the high frequency difficulties encountered, and I am grateful to the members of the department of physics for their interest and co-operation, most especially to Professor A. P. Wills for his constant encouragement, advice, and criticism.

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COLUMBIA UNIVERSITY,
April 27, 1923.

TRANSFORMATION OF ELECTRIC AND MAGNETIC FORCES IN A PLANE WAVE, IN A PLANE NORMAL TO THE DIRECTION OF RELATIVE MOTION OF TWO OBSERVERS

By Vladimir Karapetoff

Abstract

Two observers are assumed to move along the X axis and to measure the components of the electric intensity $E(E_1, E_2, E_3)$ and magnetic intensity $H(H_1, H_2, H_3)$ of a plane electromagnetic wave propagated at an arbitrary angle to the X axis. According to the theory of relativity, the two observers will find the same values of the components E_1 and H_1 along the X axis. The components E_2, E_3, H_2, H_3, along the Y and Z axes, are replotted in a novel manner in the so-called *computing plane* which consists of two sets of oblique co-ordinates. The angle comprised by each set is uniquely determined by the relative velocity of the two observers. In the computing plane a transformation point M is located such that its abscissa and ordinate with respect to one set of oblique axes gives the components E_2 and H_3 of the wave intensities as measured by one of the observers, while its co-ordinates with respect to the other set of axes gives the values of the same components as measured by the other observer. Another transformation point N gives the components E_3 and H_2 for the two observers. It is shown that the loci of M and N are ellipses. The results obtained are shown to check with the well-known equations of the theory of relativity. The location of the points M and N is shown in two simple special cases, and instructions are given for locating them in the general case.

CONSIDER an observer S, with an orthogonal right-handed system of axes XYZ which are stationary with respect to himself, and let a plane electromagnetic wave be propagated in some direction which does not coincide with any of the coordinate axes. Let a second observer S' be in uniform motion with respect to S, in the positive direction of the X axis, and let the observer S' refer his observations to an orthogonal right-handed system of axes $X'Y'Z'$ moving with him, and such that the X' axis coincides with the X axis while the Y' and Z' axes remain parallel to the corresponding axes Y and Z of the first observer. Furthermore, let the velocity of light in vacuum be taken as unity, and the relative velocity q of the two observers be expressed as a fraction of the velocity of light.

The S observer determines the components, parallel to the coordinate axes, of the electric and magnetic intensities (E and H) of the wave, and so does the S' observer. Let the S observer find the amplitude values of the

components of the electric intensity parallel to the X, Y, and Z axes to be E_1, E_2, E_3 respectively and let the corresponding values of the magnetic intensity be H_1, H_2, H_3. Let the S' observer measure the values E_1', E_2', E_3', H_1', H_2', H_3'. According to the restricted theory of relativity, we have the following relations among these quantities:[1]

$$E_1' = E_1 \tag{1}$$

$$E_2' = \beta(E_2 - q\,H_3) \tag{2}$$

$$E_3' = \beta(E_3 + q\,H_2) \tag{3}$$

and also

$$H_1' = H_1 \tag{4}$$

$$H_2' = \beta(H_2 + q\,E_3) \tag{5}$$

$$H_3' = \beta(H_3 - q\,E_2) \tag{6}$$

where

$$\beta = (1 - q^2)^{-\frac{1}{2}} \tag{7}$$

The sinusoidal function which expresses variations of E' and H' with the time is here omitted, since we are concerned only with their amplitudes; the questions of aberration and Doppler effect are not considered in this article. The two observers find the same values of the electric and magnetic components in the direction of their relative motion, but different values of the Y and Z components.

The purpose of the present article is to show that Eqs. (2), (3), (5), and (6) admit of a simple graphical interpretation. The diagrams shown below not only permit to visualize these relationships and to check the signs of the individual components, but may lead to new relationships not apparent in the analytical equations.

In Fig. 1 two sets of *oblique axes* are shown, viz., YOZ and $Y'OZ'$. This figure and the axes are intended only for computation purposes, that is, for plotting the values of the electric and magnetic components, and have nothing to do with the actual relations in space. In other words, Fig. 1 represents an arbitrary plane for a graphical representation of the above-mentioned equations, and we shall refer to it as the *computing* plane. The two sets of axes are obtained as follows. Let OC be an arbitrary reference axis, and OY_0 and OZ_0 two mutually perpendicular directions at 45° to OC. Then the YOZ set of axes is obtained by expanding Y_0OZ_0 symmetrically by an angle α, while the $Y'OZ'$ set is obtained by contracting the Y_0OZ_0 set by the same angle α. OC is the common bisector for all the three sets of axes. The acute angle α is determined by the relationship

$$\sin \alpha = q \tag{8}$$

[1] See, for example, O. W. Richardson, *The Electron Theory of Matter*, 1916, p. 309.

where q, as mentioned above, is the relative velocity of the two observers, expressed as a fraction of the velocity of light. Eq. (7) becomes

$$\beta = \sec \alpha \tag{9}$$

We shall now prove that *for any given plane electromagnetic wave and a given relative velocity q of the two observers, there are in the computing plane two transformation points, M and N, equivalent to the Eqs. (2), (3), (5), and*

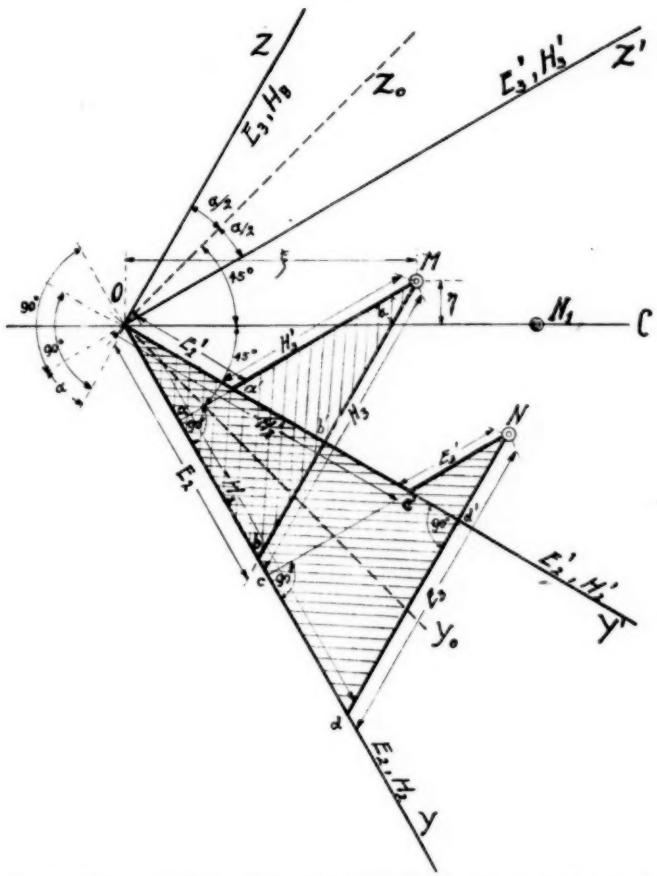

Fig. 1. The computing plane with transformation points M and N.

(6). To obtain point M, we plot along OY the value $Ob = E_2$, measured by the S observer, and draw the ordinate $bM = H_3$, parallel to the OZ axis. From the geometry of the figure, the abscissa Oa' of the same point M in the $Y'OZ'$ system is equal to the electric component E_2' measured by the S' observer, and the ordinate $a'M$ is equal to the magnetic component H_3' found by the same observer. Namely,

$$E_2' \cos \alpha = E_2 - H_3 \sin \alpha \tag{10}$$

Multiplying both sides of this equation by sec α, gives

$$E_2' = \sec \alpha \, (E_2 - H_3 \sin \alpha) \tag{11}$$

Substituting for sin α and sec α their values from Eqs. (8) and (9), Eq. (2) is obtained. Similarly, from Fig. 1,

$$H_3' \cos \alpha = H_3 - E_2 \sin \alpha \tag{12}$$

Multiplying both sides of this expression by sec α and using again Eqs. (8) and (9), an expression is obtained, identical with Eq. (6).

The transformation point N in the YOZ system has the abscissa equal to $-H_2$ and the ordinate E_3. The necessity for a minus sign before H_2 will be seen by considering an electromagnetic wave propagated along the X axis, with the electric intensity in the first YZ quadrant. Three of the projections, E_2, E_3, and H_3, are then positive, while H_2 is negative. Since in Fig. 1 both transformation points, M and N, are shown in the first quadrant, H_2 must be taken with the minus sign.

To prove that the transformation point N represents Eqs. (3) and (5), we write, from Fig. 1

$$E_3' \cos \alpha = E_3 + H_2 \sin \alpha \tag{13}$$

$$-H_2' \cos \alpha = -H_2 - E_3 \sin \alpha \tag{14}$$

Multiplying these equations by sec α, and using Eqs. (8) and (9), the foregoing expressions are converted into Eqs. (3) and (5).

We thus see that the relations (2), (3), (5), and (6) can be represented by the cross-hatched quadrilaterals $Oa'Mb$ and $Oc'Nd$. It is understood of course that in reality H_2 is normal to H_3 and E_2 is normal to E_2, and that the computing plane is to be used only for transforming the individual components from one observer to the other and not for the total values of E and H. There is nothing in Fig. 1 to indicate that one of the observers is "stationary" and the other "moving." The diagram merely states that if one of the observers measures certain values, say E_2 and H_3, then the observer whose *relative* velocity is q in the positive direction of the X axis, will find the values E_2' and H_3'.

It is of interest to find the locus of the transformation points M and N when the relative velocity q or (which is the same) the angle α varies. Since the position of the bisector OC remains the same, it is natural to orient M and N with respect to OC. Let the orthogonal co-ordinates of M be ξ and η. We then have

$$\xi = (H_3 + E_2) \cos (45^\circ + 0.5\alpha) \tag{15}$$

$$\eta = (H_3 - E_2) \sin (45^\circ + 0.5\alpha) \tag{16}$$

Eliminating α from these two equations gives an ellipse

$$\xi^2/(H_3 + E_2)^2 + \eta^2/(H_3 - E_2)^2 = 1 \tag{17}$$

A similar equation may be derived for the locus of the transformation point N.

Two Special Cases

(a) Let us consider the simplest possible case of a plane wave propagated along the X axis, with the electric intensity along the Z axis and the magnetic intensity along the negative Y axis. Let the magnitudes of both intensities be A. Then $E_1=E_2=0$; $H_1=H_3=0$; $E_3=A$; $H_2=-A$. In the computing plane (Fig. 1) the transformation point M coincides with the origin O, and the point N (denoted by N_1) lies on the bisector OC. It will be seen from the sketch that $E_2'=H_3'=0$ and

$$E_3' \cos (45°-0.5a)=E_3 \cos (45°+0.5a) \tag{18}$$

from which

$$E_3'=-H_2'=A \tan (45°-0.5a)=A\sqrt{(1-q)/(1+q)} \tag{19}$$

This shows the apparent reduction in the magnitude of the two intensities for an observer traveling at a velocity q in the direction of propagation of the wave. The elliptic locus degenerates into the straight line ON'.

(b) The example considered by Richardson[1] is shown in the computing plane in Fig. 2. The direction of the propagation of the wave is in the XY plane, at an angle θ to the X-axis. The angle θ is considered positive towards the negative Y axis. The electric intensity is in the direction of

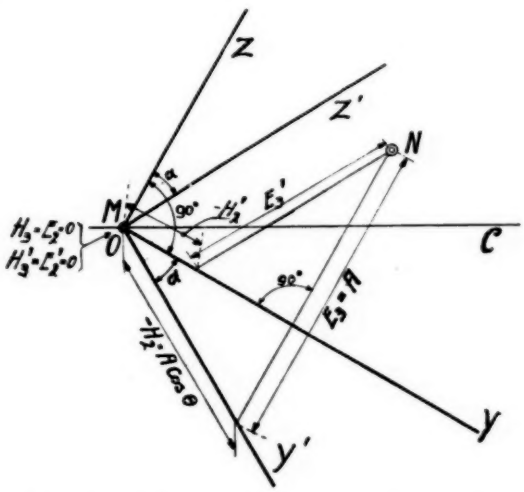

Fig. 2. Transformation of the E and H components of a wave propagated at an angle θ to the X axis, E being directed along the Z axis.

the Z axis. Here we have $E_1=E_2=0$; $E_3=A$; $H_1=-A \sin \theta$; $H_2=-A \cos \theta$; $H_3=0$. The transformation point M again coincides with the origin; the transformation point N lies above the bisector OC. We see from the diagram that $H_3'=E_2'=0$; the quantities E_3' and H_2' will also be found to agree with the expressions given by Richardson.

The Co-ordinates of M and N in the General Case

Let l, m, n be the direction cosines of the line of propagation of the wave, let its electric intensity be defined by its direction cosines l_e, m_e, n_e, and let the magnetic force H be similarly determined by the direction cosines l_h, m_h, n_h. Since the direction of propagation and the directions of E and H form three mutually perpendicular lines, we have:[2]

$$\begin{aligned} ll_e+mm_e+nn_e&=0\\ ll_h+mm_h+nn_h&=0\\ l_el_h+m_em_h+n_en_h&=0 \end{aligned} \tag{20}$$

and also the usual parallelopiped conditions

$$\begin{aligned} l^2+m^2+n^2&=1\\ l_e^2+m_e^2+n_e^2&=1\\ l_h^2+m_h^2+n_h^2&=1 \end{aligned} \tag{21}$$

A wave is completely determined by four out of the nine direction cosines and by its intensity $E=H=A$. The other direction cosines can be determined from Eqs. (20) and (21). Thus, for a given wave we may write

$$E_1=Al_e;\quad H_1=Al_h; \tag{22}$$

$$E_2=Am_e;\ H_3=An_h; \tag{23}$$

$$E_3=An_e;\ H_2=Am_h. \tag{24}$$

The values given by Eqs. (22) are the same for the S and S' observers. The values given by Eqs. (23) determine the transformation point M (Fig. 1) and those given by Eqs. (24) determine the transformation point N. It must be remembered that the abscissa of the point N is equal to H_2 with the minus sign. Having located M and N, we draw Ma' and Nc' parallel to Z'. This will give the values of the electric and magnetic intensities for the S' observer.

School of Electrical Engineering,
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September 11, 1923.

[2] See, for example, Snyder and Sisam, Analytic Geometry of Space, Chap. III.

A NEW APPLICATION OF THE BAR METHOD FOR THE MEASUREMENT OF THERMAL CONDUCTIVITY

By Marcus D. O'Day

Abstract

Thermal conductivity.—(1) *Electrically heated bar method of measurement.* Equations are derived for the case of a long thermally insulated bar, with ends kept at the same constant temperature, and carrying an alternating current such as to make the temperature distribution parabolic. This is possible only with conductors for which $(\alpha-3\beta)>0$, where α and β are the temperature coefficients of electrical and thermal conductivity, respectively. It is shown that the conductivity at each end $K_\theta=jI^2R_0L^2/2A\theta_m=jI^2R_0L/Ap_0$, where jI^2R_0 is the heat generated per unit length at the ends, $2L$ is the length, A is the cross section, θ_m is the mean temperature above that at the ends, and p_0 is the temperature gradient at the ends. (2) *Conductivity of lead and tin.* A long test bar was used, with ends fastened into heavy copper blocks, insulated from each other but both in the same thermostatic oil bath. The temperature distribution was determined by thermo-junctions and the critical current found by interpolation. For lead, $K_0=.0877$, $\beta=.000138$; for tin $K_0=.1575$, $\beta=.00067$.

Thomson effect.—(1) *Bar method.* In connection with thermal conductivity measurements, the coefficient may be determined from observations of the effect at a point $.42L$ from one end, of reversing a direct current of the critical magnitude. (2) *The coefficient for tin* at 38° came at 9.7×10^{-7} cal. per coulomb per sec.

THE writer restricts the term "bar method" to mean one of the electrical methods such as used by Jaeger and Diesselhorst,[1] and

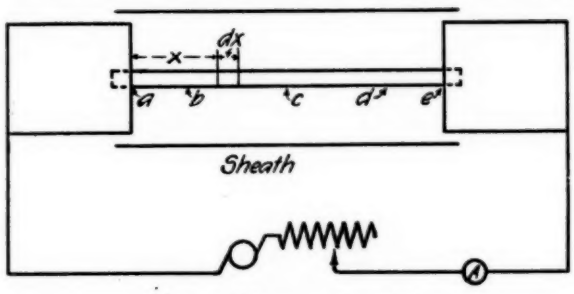


Fig. 1. Diagram of method.

Callendar.[2] The principle of such a method is illustrated in Fig. 1. A bar of metal AC is placed between two heavy copper blocks kept at a

[1] Jaeger and Diesselhorst, Preusse. Akad. Wiss., Berlin, **38**, 711, 1899

[2] Callendar, Article on Conduction of Heat in Encyclopedia Britannica

constant temperature. It is covered with some fairly good heat insulating substance and then by a metal sheath, electrically insulated from the two copper blocks. Thermo-couples are placed along the bar for the measurement of the temperature distribution when an electric current is flowing through it. The whole is immersed in a constant temperature bath, the temperature of which is taken as a zero of reference. We can measure directly the following quantities

The dimensions of the bar; the length $2L$ and the cross section A;

The resistance of the bar $R_0(1+\alpha\theta)$ where R_0 is the resistance of the bar at the zero of temperature.

The temperature gradient at the ends p_0;

The temperature at the middle θ_m;

The electric current passing through the bar I.

The quantities which we cannot measure directly are:

The heat conductivity of the bar, $K=K_0(1-\beta\theta)$;

The heat conductivity of the insulating substance, $k=k_0(1-\gamma\theta)$;

The coefficients β and γ;

The emissivity of the bar E.

THEORY

The amount of heat crossing a plane section of the bar at a distance x due to conduction along in time dt is,

$$Q_1=AK\ (d\theta/dx)dt$$

The amount of heat accumulating in the element in time dt is,

$$Q_1-Q_2=A\frac{d}{dx}\left(K\frac{d\theta}{dx}\right)dxdt$$

The amount of heat accumulating in the element in time dt due to the Joule heating of the electric current is $jI^2R_0(1+\alpha\theta)dxdt$ where $j=.239$, the number of calories in a joule. The amount of heat accumulating in the element in time dt due to the Thomson effect, "Specific heat of Electricity," which is directly proportional to the current and to the temperature gradient is $-Is\ (d\theta/dx)dxdt$ where s is the Thomson coefficient.

For the steady state the sum of these three terms must be equal to the loss of heat through the surface, which is, assuming Newton's law of cooling $h\theta dxdt$ where h is Ep, the emissivity times the perimeter of the bar.

We thus get as the differential equation expressing the flow of heat in a rod carrying an electric current

$$A\frac{d}{dx}\left(K\frac{d\theta}{dx}\right)-Is\frac{d\theta}{dx}+(jI^2R_0\alpha-h)\ \theta=-jI^2R_0 \qquad (1)$$

This equation is of the form

$$\frac{d}{dx}\left(K\frac{d\theta}{dx}\right)-a\frac{d\theta}{dx}+b\theta=-c \qquad (1')$$

The integration of this equation in the general case is not feasible. There are certain special cases, however, of great interest from the experimental standpoint, where the equation reduces to one with a simple algebraic integral.

Case I. In this case we use an alternating current so as to make a zero and so adjust its value that the quantity b is also made zero. This requires that

$$h = jI^2R_0a \tag{2}$$

This was first suggested by Callendar.[2] Putting for K its value $K_0(1-\beta\theta)$, the second integral of the reduced equation is seen to be

$$\theta K_0(1-\tfrac{1}{2}\beta\theta) = -\tfrac{1}{2}cx^2+Dx+E$$

We have as our boundary conditions, $\theta=0$ when $x=0$ and when $x=2L$, whence

$$E=0, \text{ and } D=cL \tag{3}$$

The quantity $K_0(1-\frac{1}{2}\beta\theta_m)$ is easily seen to be the heat conductivity at the temperature $\frac{1}{2}\theta_m$ and is the mean conductivity which we shall denote by K_m. We have the relation that when $\theta=\theta_m$, $x=L$ (at the middle). Hence

$$K_m = cL^2/2\theta_m = jI^2R_0L^2/2A\theta_m \tag{4}$$

We can determine the constant D by the relation that $K=K_0$ and $d\theta/dx=p_0$ when $x=0$. We thus get $D=K_0p_0$. Combining this with Eq. (3)

$$K_0 = cL/p_0 = jI^2R_0L/Ap_0 \tag{5}$$

From Eqs. (4) and (5) we can calculate the value of β, the temperature coefficient.

The question naturally arises how the experimenter knows how to adjust the current to such a value that Eq. (2) is satisfied. Probably the best way to do this is to measure the quantity h directly as was done by King.[3] If we use a very small current, the value of θ will not be large and the temperature gradient near the middle of the bar will be zero. In this case all of the heat generated at this portion of the bar will be lost and we have the relation

$$h\theta_m = jI^2R_m \tag{6}$$

Case II. In this case we adjust the current until we have a perfect parabolic distribution of temperature. We will use alternating current so as to make the second term zero. We can write Eq. (1′)

$$K_0\frac{d^2\theta}{dx^2}-\beta K_0\theta\frac{d^2\theta}{dx^2}-K_0\beta\left(\frac{d\theta}{dx}\right)^2+b\theta+c=0 \tag{7}$$

and so adjust the current that the sum of the three middle terms is equal

[3] King, Amer. Acad. Proc. **33**, 353, 1898

to zero. To do this it is necessary to solve the following equations and eliminate θ.

$$K_0\,(d^2\theta/dx^2)+c=0 \tag{8}$$

$$\beta K_0\theta\,(d^2\theta/dx^2)+K_0\beta\,(d\theta/dx)^2-b\theta=0 \tag{9}$$

Solving the last one first we get

$$\tfrac{1}{2}\beta K_0\,(\theta\,d\theta/dx)^2=\tfrac{1}{3}\,b\theta^3+D \tag{10}$$

Since $d\theta/dx$ is finite when $\theta=0$ we have $D=0$. Eq. (10) then integrates into

$$\sqrt{3\beta\,K_0\theta}=\sqrt{\tfrac{1}{2}b}\,x+E$$

and since $x=0$ when $\theta=0$, we have $E=0$. Squaring we get

$$3\beta\,K_0\theta=\tfrac{1}{2}\,bx^2 \text{ and } \theta=\theta_m \text{ when } x=L \tag{11}$$

It will be shown later from Eq. (8) that $2K_0\theta_m=cL^2$ which substituted into Eq. (11) gives

$$b=3\beta c \tag{12}$$

Replacing b and c by their values from Eq. (1) we get

$$jI^2R_0\alpha-h=3\beta jI^2R_0, \quad I^2=h/jR_0(\alpha-3\beta) \tag{13}$$

The larger the difference, $\alpha-3\beta$, the better the method and it fails when this difference is very small or negative.

The first integral of Eq. (8) for the critical value of the current is $K_0\,(d\theta/dx)=-cx+D$ and the second integral is

$$K_0\theta=-\tfrac{1}{2}\,cx^2+Dx+D' \tag{15}$$

which reduces to

$$K_0=jI^2R_0L^2/2a\theta_m \tag{16}$$

if we apply the same boundary conditions as before. This replaces Eq. (4) and instead of Eq. (5) we get

$$K_0=jI^2R_0L/A\;p_0 \tag{17}$$

These two equations are compatible only when

$$\theta_m=\tfrac{1}{2}L\;p_0 \tag{18}$$

The above relation shows how we may realize the conditions of Eq. (13) without knowing either h or β. If the bar under investigation is of reasonable length (25 cm or over) a thermo-couple placed one centimeter from the ends of the bar will give the value of p_0. A thermo-couple is placed at the end of the bar as in Fig. (1) to get the temperature of the end point. The experimenter need only to try several different currents, plot the variations from Eq. (18) and note at what current the relation is satisfied. If he does this for two different temperatures of the constant temperature bath, he can calculate β, and then from Eq. (16) he may obtain h.

If h is known and the inner and outer radii R' and R of the insulating layer, the heat conductivity of the insulating substance may be calculated from the formula

$$K=(h/2\pi)\,\log\,(R/R') \tag{18'}$$

Both methods were used by the writer, the first on lead and the second on tin. However, he prefers the latter method especially when $\alpha-3\beta$ is fairly large. The difficulties usually met with, viz., the measurement of small temperatures and the loss of heat through the insulating substance, have to a large extent been overcome. We have a very accurate method of measuring h, and the temperature differences are very large, the smallest being about five centigrade degrees. The measurement of so large a temperature presents no great experimental difficulties.

After the experimenter has found the correct value of the current he may use direct current of the same value and measure the Thomson effect. Instead of Eq. (8) we have

$$K_0\frac{d^2\theta}{dx^2}-a\frac{d\theta}{dx}=-c \tag{19}$$

which has for its integral

$$\theta=Be^{mx}-D+cx/m \tag{20}$$

where $m=a/K_0$.

Using the same boundary conditions we get

$$\theta=\frac{c}{m}\{x-2L\ (e^{mx}-1)/e^{2mL}-1)\} \tag{21}$$

and

$$B=D=-2cL/m(e^{2mL}-1) \tag{22}$$

The second term of the right hand side of Eq. (21) can be expanded by means of Bernoulli's functions, and neglecting m^2 in comparison with m, since it is less than 10^{-5}, we get

$$\theta=c\{x(2L-x)+\tfrac{1}{6}\ mx(2L-x)\ (L-X)\} \tag{23}$$

The value of m changes sign with the direction of the current and calling $d\theta$ the change in temperature obtained upon reversal of the current, we get

$$d\theta=\tfrac{1}{3}\ cmx(2L-X)\ (L-X)$$

This is a maximum when $x=.42L$. Therefore if we place a thermo-element at this point and measure $d\theta$, we get for the Thomson coefficient

$$s=A^2K_0d\theta/0.128\ jI^3L^3R_0 \tag{25}$$

Since the value of the current is determined by Eq. (13), the only arbitrary quantity in the above equation is the length $2L$ and the equation shows that increasing the length very greatly increases the sensibility. The Thomson coefficient is measured in calories per coulomb per sec. and is of the order of 10^{-7}. It is thus seen that $d\theta$ is necessarily a very small difference in temperature. Ordinarily currents of from three to ten amperes are used, while in this method the currents are well above fifty amperes. Since $d\theta$ is proportional to I^3 in the latter case it will be very materially increased. If the loss of heat from the surface is not compensated, the error caused thereby is proportional to the length of the bar.

In this case where it is compensated, the length can be as large as mechanically feasible. Eq. (25) shows that increasing the length has the same effect as increasing the current.

The method differs from that used by the majority of investigators in that the ends are kept at the same temperature and the heating of

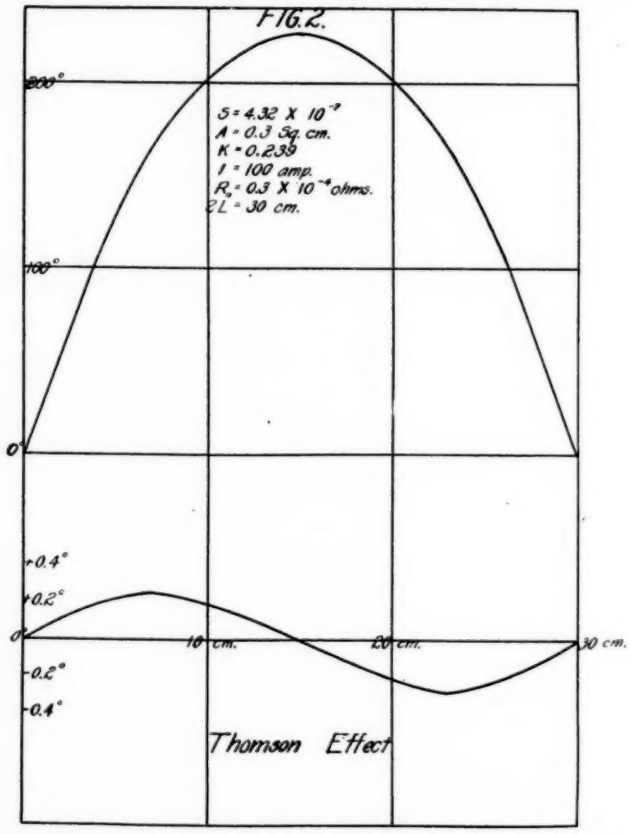


Fig. 2. Upper curve: Temperature distribution for ideal case. Lower curve: Effect of Thomson effect on the distribution.

the current causes its own temperature gradient. In Fig. 2, the writer has plotted the temperature distribution for an ideal case which is approximately that of tin. It is seen that for a given maximum temperature the method outlined above gives a much larger temperature gradient. Since the value of a in Eq. (19) is directly proportional to the temperature gradient, it is seen that $d\theta$ is much greater in the method suggested. The bottom curve in Fig. (2) (drawn on a scale one hundred times as large) shows the effect of the Thomson heating upon the temperature distribution.

Description of Apparatus

The bar of metal to be investigated was screwed into two large copper blocks, two inches in diameter and one foot long. The bar was bent in the shape of a U, as in Fig. 3. The copper blocks were separated by thin sheets of mica. The bar was covered with wool yarn and placed in an inverted Dewar flask. The copper blocks were then placed in an oil

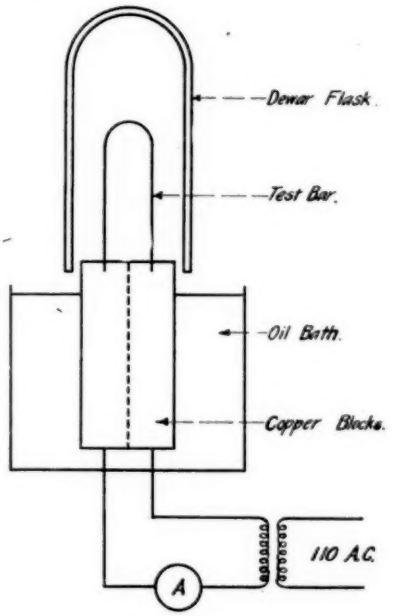


Fig. 3. Apparatus.

bath kept in motion by a stirrer. The alternating current was supplied by a low voltage transformer and measured by a current transformer. Five thermo-couples were attached to the bar and from these it was easy to get p_0 and θ_m. For the measurement of p_0 the thermo-couples were placed two centimeters apart. The thermo-couples were then brought to a special selector switch which enabled a series of thermo-couples to be balanced very rapidly against one standard thermo-couple. This switch was so designed as to eliminate spurious e.m.fs. at the contact points. A Leeds and Northrup type K potentiometer was used for measurement of temperature with the standard cell replaced by a standard thermo-couple with one junction in ice and the other in steam. This proved to be very satisfactory and enabled the potentiometer to read directly in degrees as thermo-junctions were chosen to all have the same e.m.f. Thermo-couples soldered without twisting proved to be much more

uniform than twisted ones. Isolated thermo-couples were not used in the case of lead and the Thomson effect was not measured. However, in the case of tin, thermo-couples coated with lacquer and then glued into the bar gave good results and enabled a determination of the Thomson effect.

Data on Lead

A bar of lead, supposed to be Kahlbaum's, 25 cm long, was used. Only K and β were determined in these early experiments as the apparatus was designed before the complete theory of the method was attempted. Also the Thomson effect was not measured, it being difficult to eliminate leaks in the potentiometer circuit. The method used was that of Case I. The data taken with the bath at room temperature are as follows: $I=50$ amp; $L=12.5$ cm; $A=0.439$ cm^2; $R_0=0.000478$ ohms (resistance at 20°C); $c=0.6504$.

Table I

Results for lead at room temperature

Temp. (ends)	p_0	θ_m	K/c	K_0/c	K_m/c
25.97	8.72	52.34	0.1370	0.1430	0.1490
26.67	8.91	53.14	0.1350	0.1400	0.1450
27.14	8.90	53.76	0.1355	0.1406	0.1455
26.47	8.87	52.93	0.1345	0.1409	0.1475
Means: 26.56	8.85	53.17	0.1355	0.1411	0.1467

From this the heat conductivity (in c.g.s. units) was found to be

At 0° C	0.0877
At 26.5°C	0.0920
At 53.1°C	0.0955

By using values of heat conductivity for wool given in the tables, h was calculated by use of Eq. (18′). From this it was estimated that 50 amperes would approximately compensate for the loss of heat through the surface. In Fig. 4 are given the values of the conductivity at these three points and the straight line drawn. It is to be noted that the value of the conductivity at these three points is from one set of readings at just one temperature of the bath. The bath was then heated to varying temperatures and the conductivity at these temperatures noted. In this case the constant c was not equal to jI^2R_0 but to $(jI^2R_0+h\theta_0)$ where θ_0 is the temperature of the surrounding atmosphere. Since it had a large negative value, measurements were difficult and values of the temperature gradients were not consistent so the heat conductivity was calculated from the formula

$$AK_m=(c_1-c_2)^2L^2/\Delta\theta_m=jI^2R_0/\Delta\theta_m \qquad (28)$$

where $\Delta\theta_m$ is the difference in the values of θ_m when the current is flowing and when it is shut off. Readings were taken with the current off for

thirty minutes, on for thrity minutes, and off again for thirty minutes. This gave a good average to substitute in the above formula. Since the resistivity of the bar checked exactly with that given by the Bureau of Standards, their value of the temperature coefficient was used.

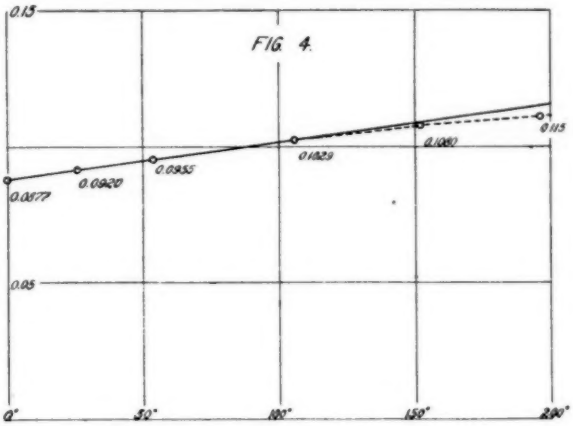


Fig. 4. Thermal conductivity of lead as a function of temperature.

Data on Tin

The method of Case II was used. In the following table let V be the per cent variation from the true parabolic distribution of temperature $2L=28$ cm; $A=0.211$ cm^2; $R_0=0.000528$ ohms per cm (room temp.); $\alpha=0.0042$; and $c=0.584\ I^2\times10^{-3}$.

I	Temp. (ends)	p_0	θ_m	$\theta_m-\frac{1}{2}Lp_0$	V	$\sqrt{V}$
15 amp.	20.52	0.970	6.15	−0.640	10.40	3.20
20	20.21	1.705	10.90	−1.035	9.50	3.08
25	21.25	2.775	17.48	−1.945	10.90	3.30
30	22.24	3.855	24.51	−2.490	10.05	3.16
35	22.43	5.475	35.24	−3.069	8.69	2.95
40	23.57	7.630	49.97	−3.430	6.78	2.60
45	26.00	10.270	68.50	−3.090	4.50	2.12
50	29.14	13.780	96.46	+1.040	−1.07	1.03

In Fig. 5 two curves are plotted giving the manner in which $\sqrt{V}$ and p_0 depend upon the current. It is evident from the figure that the balancing current is 49.2 amperes and that the value of p_0 is 13.00. This gives a value of the conductivity equal to 0.1575 c.g.s. units. The peculiar behavior of the curve in the neighborhood of 20 amperes may be due to a recalescence phenomenon which the writer hopes to investigate at a later date. In computing the value of the conductivity it is to be noted that R_0 has increased, owing to the fact that the ends have been raised to a higher temperature. This is also to be noted in the calculation of the

temperature coefficient β. The value of h was determined from Eq. (6) and found to be $h=(2.64)^2\times5.28\times10^{-5}/0.448\times4.184=1.96\times10^{-4}$ cal./sec., which combined with Eq. (16) gives for β the value -0.000670.

A direct current of more than 36 amperes which was sufficiently steady for the measurement of the Thomson effect was not available. However with a current of 36.1 amperes, there was a deflection corresponding to 0.4° C upon reversal of the current. This value of $d\theta$ substituted in Eq. (25) gives $s=9.7\times10^{-7}$ calories per coulomb per sec. This would be the value of the Thomson effect at 38°.

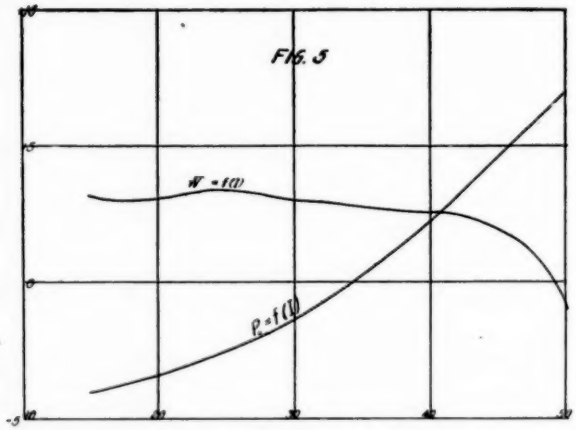


Fig. 5. Case II. Variation of $\sqrt{V}$ and p_0 with current I.

The results show that the samples were not of the utmost purity as the temperature coefficients are positive. The writer hopes to investigate this point further in later researches on the effect of a systematic alloying of the lower melting point metals upon the thermal conductivity. He wishes to express his gratitude to Professor E. E. Hall, of this university whose interest and suggestions have been the inspiration of this work. He is also indebted to Professors Boynton and Caswell of the University of Oregon for valuable suggestions concerning the Thomson effect.

UNIVERSITY OF CALIFORNIA,
DEPT. OF PHYSICS,
August 13, 1923.

THE SIMPLE RIGIDITY OF A DRAWN TUNGSTEN WIRE AT INCANDESCENT TEMPERATURES

By William Schriever

Abstract

Rigidity of a 10-mil drawn tungsten wire to 2000°K.—To avoid errors due to end effects, two lengths of the same wire were mounted in series and heated by the same current so that the middle portions of both were at the same temperature. The apparatus was placed in an evacuated tube; the torque was applied magnetically and measured by the twist in a cold fine tungsten wire. The rotations of the mirrors attached to the ends of the heated wires were noted within a few seconds after application of the torque, since at the higher temperatures even a twist of 0.1° per cm exceeded the elastic limit. On the three successive runs made, the rigidity increased progressively due to the crystal growth in the temperature range above 1600°K. For the recrystallized or equiaxed wire the values of the modulus obtained on the third run are: 21.7 at 300°K, 21.0 at 1030°K and 3.4 at 1985°K, all times 10^{11} dynes/cm^2.

Introduction

UP to the present time, as far as is known to the writer, the simple rigidity of a drawn tungsten wire at incandescent temperatures has not been determined.

Koch and Dannecker[1] seem to have made the first determination of the simple rigidity of a metal at temperatures which approach the melting point of the material; they made observations on wires of some twenty-two elements and alloys using the usual torsion-pendulum method. They allowed for the changes in periods caused by the temperature gradients along the wires, by making several assumptions and approximations together with graphical interpolations and extrapolations—a very indirect method.

When the temperatures desired are not too high the whole apparatus may be heated in some sort of an oven and the temperature gradients in the wires, the "end-effects," thereby avoided. Since an apparatus which could be heated to 2000°K did not seem feasible, the following simple method of allowing for the end-effects was devised. The same torque was applied to two different lengths of the same piece of electrically heated wire whose central portions were at the same temperature. When the angle of twist of the shorter was subtracted from that of the

[1] Koch and Dannecker, Ann. der Phy. **47**, 197-227, 1915

longer the difference was the angle of twist produced by that torque in a uniformly heated wire of a length equal to the difference of the lengths of the two wires. This scheme necessitated using a static method for making the simple rigidity determinations. Since glowing tungsten is oxidized in the open air or in the presence of water vapour, the wires and attached apparatus were enclosed in an evacuated vessel.

Streintz[2] and Meissner[3] have found that the simple rigidities of all the metals which they investigated are independent of the manner of heating. While it is possible, it is not probable that tungsten wire would show a difference. In fact Dodge[4] has found that Young's modulus of tungsten wire is independent of the method of heating between 0° and 1000°C. The writer has thus far not attempted to check this point for higher temperatures experimentally.

Apparatus and Procedure

A schematic drawing of the wires and attached apparatus, all of which was enclosed in a large glass tube, is shown in Fig. 1. The short length and the long length of the tungsten wire on which the simple rigidity determinations were to be made are shown at L_1 and L_2 respectively. L_3 is a relatively fine tungsten wire attached by means of a clamp to the lower end of L_2, the axes of the wires being in the same straight line; L_3 was never heated and was used to measure the torque applied to L_1 and L_2. The top end of L_1 was clamped securely in the end of the vertical round brass rod to which were attached the soft iron cross-bar *Fe* and the mirror M_1. An electro-magnet acting through the glass tube on the iron cross-bar made it possible to rotate the rod and therefore to twist the series of wires.

The lower end of L_3 was held securely in a clamp to which were attached the mirror M_4 and the weight W; the clamp had a rectangular cross section and could thus slide up and down, as the wires changed in length due to thermal changes, but could not turn. The weight of the clamp and W (170g) served to hold the wires taut.

To the clamp which held together L_2 and L_3 was attached the mirror M_3 and an inverted U of iron wire which dipped into the mercury in the annular iron mercury-cup *Hg*. The mercury-cup and the parts of the brass frame were so insulated that the heating current flowed in at P_1, down R_1 to *Hg*, up L_2 and L_1, and out at P_2. L_1 and L_2 were one continuous piece of tungsten wire separated into two segments by a brass clamp to which was attached the mirror M_2.

[2] Streintz, Pogg. Ann. **150**, 368-380, 1873
[3] Meissner, Ann. der Phys. Beibl. **34**, 756, 1910
[4] Dodge, Phys. Rev. **11**, 311, 1918

Each mirror was made by depositing a film of platinum on a cover-glass by an evaporation method, covering the film side with another cover-glass, and mounting the two in a light copper frame. Silvered mirrors put together in the same manner were tried first, but the mercury vapor soon caused them to lose their reflecting power almost completely, while the platinized mirrors seemed to be in perfect condition after several months of exposure to mercury vapor. The asbestos tape which was wound around the three cross-heads to make the frame fit the glass tube snugly, had previously been heated to 350° C in a tube attached to a vacuum pump, in order to get rid of the organic matter which would otherwise discolor the glass tube of the final apparatus and thus make optical temperature measurements inaccurate.

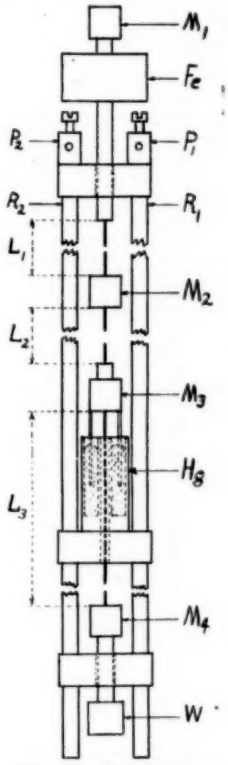


Fig. 1. Torsion system for determining the rigidity.

After the frame had been placed in the tube and the mirrors were in proper adjustment, annealed coiled copper leads were pulled out of side-arms and made fast in the binding posts P_1 and P_2. The top of the large tube was then closed by sealing on a cap to which was attached a pump connection. A side-tube of small diameter was sealed into the large tube so that mercury could be let into the mercury-cup through it. This apparatus was placed in a long electrically heated oven from which protruded the pump connection and the small side-tube to which was attached a bulb containing sufficient clean air-free mercury. A drying-tube containing phosphorus pentoxide was placed in the pump-line. The vacuum pump was operated for about eight hours during which time the oven was maintained at a temperature slightly above 325° C. After a side discharge-tube had indicated an x-ray vacuum for three hours, the apparatus was sealed off. As soon as the apparatus was cool enough to be handled, it was removed from the oven, set in a vertical position and the mercury cup filled. The side-tube was sealed off at once thus leaving the wires in a sealed glass tube.

The scales on which the angles of twist of the mirrors M_1, and M_2 and M_3 were observed, were complete circles 360 cm in circumference, while the scale for M_4 was a short arc having the same radius. For each mirror was provided an illuminated wire and a lens for focusing its image

on its scale. Thus for each degree of twist of a mirror the image of the wire moved along the scale 2 cm.

The method of measuring the diameters of the wires has already been reported.[5] L_1, L_2 was a piece of 10-mil drawn tungsten wire of lot B24-3822 of the Nela Research Laboratory; L_3 was a piece of 6-mil tungsten wire of the same lot. Their diameters were respectively $252.3 \pm 0.3\mu$ and $156.72 \pm 0.16\mu$ and their lengths 11.91 cm, 26.62 cm and 25.03 cm at room temperature. The heating current for the 10-mil wire was furnished by large lead storage cells and was regulated by varying the number of cells and by three variable rheostats in parallel; it was measured by means of a 0.01 ohm standard resistance in conjunction with a Leeds and, Northrup potentiometer. The temperatures of the wires were measured with a Holborn-Kurlbaum type optical pyrometer; corrections were made for the absorption of the enclosing glass tube. The pyrometer used and the methods employed have already been described.[6]

The length of L_1 was such that its mid-point was at the same temperature as the mid-point of the longer section L_2 when the same heating current passed through both. Another piece of 10-mil tungsten wire 17.6 cm long, mounted in the same frame, was heated by constant currents so that its mid-point assumed the following temperatures: 1065, 1200, 1338 and 1492°K. For each mid-point temperature the temperatures of various points along the wire were obtained. The curves in Fig. 2 showing the temperature as a function of the distance along the wire measured from the top clamp, gave the information necessary to determine the minimum length for L_1.

The 10-mil tungsten wire was next heated to about 1600°K for just a few seconds in order to get rid of most of the graphite coating which always remains on a tungsten wire when its drawing is completed. Then a constant current of 4.200 amperes was passed through the wire and the brightness temperatures of the mid-points of L_1 and L_2 were found to be 1123 and 1119°K respectively. The lower section of the wire had been polished somewhat before mounting it, and consequently its graphite coating was more completely removed by the first heating; this may help to account for the observed 4° difference. At any rate the change in the simple rigidity caused by a 4° change in temperature could not have been detected since it was less than the experimental error.

The lengths of all the wires were measured and checked several times during the process of making the rigidity determinations; the measurements were always made at room temperature. The length of $(L_2 - L_1)$ at

[5] Schriever, Proc. Iowa Acad. Sci. **24**, 235-240, 1917

[6] Schriever, Proc. Iowa Acad. Sci. **28**, 69-82, 1921

a higher temperature was obtained by adding the expansion caused by the heating, the expansion being calculated by using the first term of Worthing's expansion formula.[7] Allowance was also made for the expansion of the diameter of the wire. The effects of the two expansions tended to balance in the final calculations, and if they had been neglected the error would have been small.

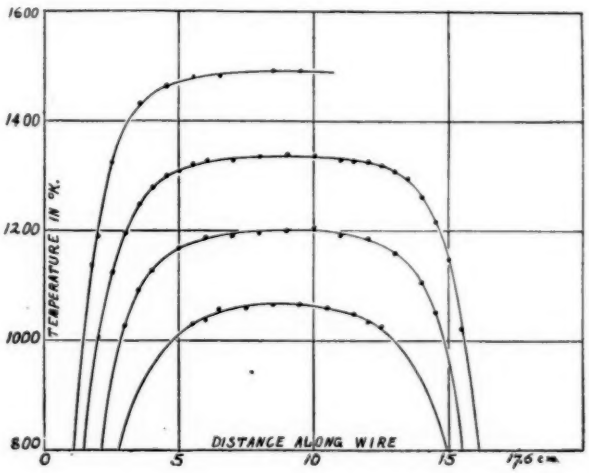


Fig. 2. Temperature distributions along a ten-mil heated tungsten filament.

After the wires L_1 and L_2 had been heated to a definite known temperature, the iron cross-bar, *Fe* in Fig. 1, was twisted through a small angle and the positions of the images reflected from the four mirrors on the four circular scales were recorded. The cross-bar was then twisted still further in the same direction and the final positions of the images were recorded. The cross-bar was then twisted back to the neutral position and the whole operation was repeated. From six to eight such sets of observations were made at each temperature. The temperature was kept constant by taking pyrometer readings at the beginning, after the third set of observations, and after the sixth set of observations, and making slight adjustments in the heating-current rheostats when necessary. At the higher temperatures the temperature was checked, and adjusted if necessary, after each set of observations. The temperature determinations were certainly correct to within 5°.

The temperature distribution along the wire at the top of L_1 was very approximately like that at the bottom of L_2, and that of the bottom end

[7] Worthing, Phys. Rev. **10**, 638, 1917

of L_1 like that at the top end of L_2. The twists in L_1 and L_2 may each be considered to have been made up of two twists, first, a twist in the end-sections in which there were temperature gradients, and second, a twist in the center section all of which was at a uniform temperature. Therefore the difference of the total twists in L_1 and L_2 gave the twist in a section of the wire of length (L_2-L_1), the temperature of all of which was that of the mid-point of either wire. Let α, β, γ, and δ be the angles turned through by the images reflected from M_1, M_2, M_3 and M_4 respectively. Then $(\alpha-\beta)=\varphi_1$ was twice the twist in L_1; $(\beta-\gamma)=\varphi_2$ was twice the twist in L_2; and $(\gamma-\delta)=\varphi_3$ was twice the twist in L_3. Therefore $(\varphi_2-\varphi_1)$ was twice the twist in a uniformly heated section of wire of length (L_2-L_1); φ_3 was twice the twist in a length L_3 of the cold wire. $\varphi_3/(\varphi_2-\varphi_1)$ obviously should be a constant for twists which do not exceed the elastic limit. The average value of this ratio for each six to eight sets of observations taken at a constant temperature was determined and its probable error calculated. $(\varphi_2-\varphi_1)$ was kept small; it was always of the order of 0.1 degrees per centimeter.

The mirrors M_2, M_3 and M_4 were displaced from the axis of rotation 0.27, 0.38 and 0.23 cm respectively. Therefore the observed angles of deflection were not twice the angles through which the mirrors were turned. Corrections for M_2 and M_3 were calculated and applied in determining the correct va'ues of β and γ for each set of observations. The deflections of M_4 never exceeded one degree and in most cases were too small to be observed.

The torque required to twist a wire of radius R and length L through an angle θ is

$$T=\tfrac{1}{2}\pi\eta\theta R^4/L$$

where η is the modulus of simple rigidity of the wire. In the apparatus employed in this work the same torque acted on the cold and hot wires. Therefore we have

$$\eta_h=\frac{\theta_c}{\theta_h}\left(\frac{R_c}{R_h}\right)^4\frac{L_h}{L_c}\ \eta_c$$

where the subscripts h and c refer to the hot and cold wires respectively. L_h/R_h^4 was calculated for each temperature, due allowance being made for the expansion as explained above. $R_c^4\eta_c/L_c$, a constant for the apparatus, was found to be 215.9 ± 0.8.* θ_c/θ_h is obviously the average $\varphi_3/(\varphi_2-\varphi_1)$, the determination of which has already been explained.

Moduli of rigidity, as given in tables of physical constants, have, in general, been obtained from angles of twist which were not great enough to

* The value of $\eta_c=14.34\pm0.04\times10^{11}$ dynes per cm^2 was obtained from Dr. L. P. Sieg; he used a static method on a piece of the 6-mil wire taken from the same spool.

cause the elastic limit of the material to be passed. In this work on tungsten the angles of twist were less than 0.1 degree per centimeter and yet, at the higher temperatures, this was considerably beyond the elastic limit. This was shown by the fact that, after the final twist was given, the mirrors M_2 and M_3 turned slowly back toward the original positions. Twists which would not strain the wire beyond the elastic limit at the highest temperatures, could not be measured accurately enough with this apparatus. In order to obtain consistent and fairly accurate results the following method of taking scale readings was used. Two card sliders to indicate the position of the beam of light reflected from a mirror, were placed on each circular scale. Four of the sliders were set at the

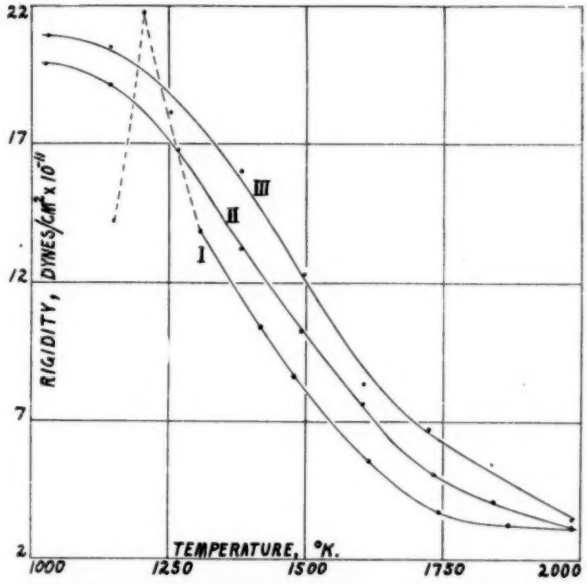


Fig. 3. Rigidity as a function of temperature for three successive runs.

initial positions of the reflected beams on the four scales. The final twist was given and the other four sliders were adjusted in the following order: first for M_3, second for M_2, third for M_4, and last for M_1. The wire was then twisted back to the initial position and the positions of the sliders were recorded. M_1 and M_4 did not change their positions, and M_2 twisted back only very slowly relative to the rate of M_3; this will explain the order of making the slider adjustments. The second slider for M_3 was in position in about two seconds after the final twist was given. The variations in the value of $\varphi_3/(\varphi_2-\varphi_1)$ for a given set at the higher temperatures, are probably largely due to the inaccuracy of the settings of the second slider for M_3.

The wire was heated for about an hour at each of the nine different temperatures of the first series, and about three quarters of an hour at each temperature of the second series. The last series was taken more rapidly; each temperature was maintained for about thirty-five minutes.

The probable error of the modulus of rigidity for each temperature was calculated from the probable error of the mean of $\varphi_3/(\varphi_2-\varphi_1)$, that of the constant of the cold wire, and that of the radius of the hot wire. The probable errors of the length-measurements were neglected since they were relatively small compared to those of the fourth powers of the radii.

Results

Curves showing the modulus of rigidity of the 10-mil drawn tungsten wire as a function of the temperature, are shown in Fig. 3. The data for each curve were obtained at a series of increasing temperatures; the curves were obtained in the order indicated by the numbers. A recapitulation of the results is shown in Table I.

Table I

Set No.	Length of hot section (cm)	R_A^4 (10^{11}cm^4)	$\Omega = 215.9L/R^4$ (10^{-10})	$\Gamma = \frac{\varphi_3}{\varphi_2-\varphi_1}$	Brightness temperature	True Temperature	$\eta_A = \Omega T$ $\left(10^{-11}\frac{\text{dynes}}{\text{cm}^2}\right)$
			Series I				
1	14.73	2576 ± 16	12.33 ± .10	11.55 ± .09	1094°K	1150°K	14.23 ± .16
2	.74	2576	.33	17.72 .43	1144	1205	21.80 .52
3	.75	2585	.33	11.28 .20	1238	1308	13.88 .25
4	.76	2585	.32	8.46 .13	1336	1418	10.41 .16
5	.77	2592	.32	7.04 .12	1389	1479	8.66 .15
6	.78	2592	.32	4.53 .23	1506	1615	5.57 .27
7	.79	2602	.29	3.01 .15	1618	1743	3.70 .18
8	.80	2608	.25	2.62 .13	1727	1870	3.22 .16
9	.81	2615	.21	2.54 .06	1823	1985	3.10 .08
			Series II				
10	14.75	2569 ± 16	12.41 ± .10	16.08 ± .06	985	1025	19.92 ± .16
11	.76	2576	.36	15.46 .25	1089	1143	19.15 .32
12	.76	2576	.36	13.57 .08	1191	1268	16.81 .16
13	.77	2585	.31	10.73 .09	1301	1380	13.21 .14
14	.78	2592	.32	8.35 .12	1401	1492	10.27 .16
15	.79	2592	.33	6.19 .03	1499	1605	7.61 .07
16	.80	2602	.30	4.15 .04	1610	1734	5.10 .06
17	.82	2608	.28	3.30 .02	1704	1843	4.06 .03
18	.86	2615	.28	2.58 .05	1823	1985	3.17 .06
			Series III				
19	14.75	2538 ± 16	12.54 ± .10	17.39 ± .14	—	296	21.73 ± .22
20	.80	2569	.45	16.84 .11	988	1030	20.98 .21
21	.81	2576	.40	16.55 .18	1089	1143	20.51 .27
22	.82	2576	.40	14.66 .14	1187	1252	18.17 .22
23	.83	2585	.38	12.94 .20	1301	1380	16.05 .27
24	.84	2592	.38	9.94 .11	1405	1497	12.31 .16
25	.85	2592	.38	6.72 .18	1499	1605	8.34 .22
26	.87	2602	.36	5.40 .15	1603	1726	6.70 .18
27	.89	2608	.31	4.45 .11	1702	1841	5.48 .13
28	.91	2615	.30	2.78 .07	1823	1985	3.42 .09

Some qualitative determinations of the simple rigidity of a 10-mil drawn tungsten wire 30.5 cm long were obtained previously.[8] No corrections were made for the end-effects but, since each temperature was calculated from the total amount of expansion, the error in the rigidity calculation tended to balance the error in the temperature calculation in such a way that the curves are more nearly correct than one might at first thought expect. The curves which were obtained are shown in Fig. 4. The data for Curve I was obtained by starting at room temperature; at the higher temperatures the elastic limit was passed so easily that observations became very inaccurate. The wire was then heated to about 2000° K for 100 minutes, allowed to cool to room temperature, and the observations for Curve II were started. Observations for Curve III were started at the highest temperature used for Curve II and completed at room temperature. A shorter length of the wire taken from the same spool gave curves having the same general shapes as those of Fig. 4, the irregularities occurring at the same temperatures.

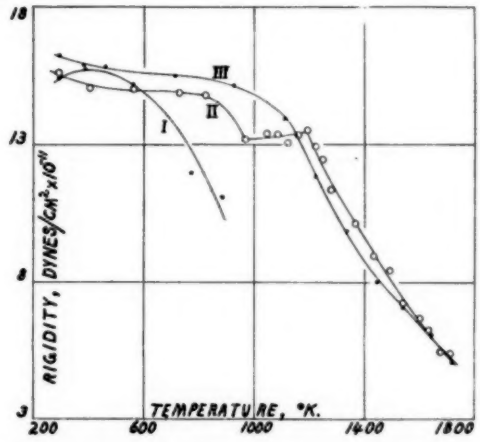


Fig. 4. Results of some preliminary measurements.

Jeffries' work[9] on the change of crystal structure of tungsten wire by heat treatments at various temperatures makes it possible to interpret the shapes of the Curves in Figs. 3 and 4. Dr. L. P. Sieg found the value 13.99×10^{11} dynes/cm² for the rigidity of the wire at room temperature; the wire had been given no heat-treatment other than that which it received when it was drawn. In the present research the temperature at which the apparatus was baked out never exceeded 330°C, and the very short heating at 1600°K (tc remove the graphite film) certainly changed the structure very little. The first rigidity determination was

[8] Schriever, Master's Thesis in Uni. of Iowa Library

[9] Jeffries, Trans. Am. Inst. Mining Eng. 1037-1092, 1918

made at 1150°K (Curve I of Fig. 3); the value was $14.2 \pm 0.3 \times 10^{11}$ dynes cm^2 which, within the experimental error, is the same as found by Dr. Sieg at room temperature. The structural changes caused by the heating at 1600°K and especially by the heat treatment at 1150°K during the process of making the first rigidity determination, will account for the constancy of the rigidity from room temperature to 1150°K for the first heating. If observations starting at room temperature had been taken, a rapid falling off of the rigidity would have been observed before a temperature of about 1100°K was reached, as was shown by the wire giving Curve I of Fig. 4.

Heat treatment at 1150 to 1300°K will cause the structure of a new wire to change rapidly. The wire (Curve I of Fig. 3) was held at 1150°K for slightly more than two hours, at 1250° for an hour, and at 1308° for about an hour. Although these temperatures are below the five-minute equiaxing or recrystallization temperature, namely 1600°K, the long exposures at these temperatures probably allowed crystallization to take place. This evidently was the case because the rigidity at 1308°K was only a little less than it was at 1150°K. From 1300 to 1700°K the curve is almost linear which indicates that no radical change in the structure took place. Above 1700°, the region of rapid grain growth, the slope of the curve decreases rapidly. Unless the crystal grains enlarged a great deal it would be difficult to explain the relatively small decrease in rigidity between 1740 and 1985°K for the first heating of the wire to such high temperatures. The fact that large crystal grains were formed would also account for the high value of the rigidity at 1025°K (first point of Curve II, Fig. 3) and again at 1030°K in the third series of measurements. Since the change of slope near the lower end of each curve is less rapid in each succeeding curve, it seems probable that, after the grain-growth is completed, the rigidity will decrease regularly with temperature-increase, and that the curves obtained thereafter will not show a decrease of slope at the higher temperatures.

If the scale readings for the final positions of M_3 at the higher temperatures had been made less rapidly, the deflections for this mirror would have been smaller, and smaller values for the modulus of rigidity would have resulted. Consequently the change of slope near the lower end of each curve cannot be due to the yielding of the wire. Thus this decreased rate of falling off of the rigidity with temperature-increase must be real and, in all probability, due to the rapid grain-growth at the higher temperatures.

It is known that the modulus of rigidity when calculated from small twists, is greater than when calculated from large twists. This effect

was observed in this research, and for this reason the twist per centimeter was kept approximately constant.

It is observed that Curve III of Fig. 4 lies below Curve II for temperatures above 1175°K. This peculiar result was probably due to the wire having formed a crystal structure which made an exceptionally rigid wire but one which was not stable at the highest temperature. The second point for Curve I of Fig. 3 is probably incorrectly located although the data do not indicate this for they are consistent among themselves.

Since in small tungsten wires a single crystal grain may occupy a very large part of the cross-section of the wire,[9] the modulus of rigidity determined from such a wire may be quite different from that obtained from observations made on a relatively large wire. An increase of rigidity with decrease in size of wire has been observed in the case of freshly drawn tungsten wires.[10] The results given in this article may therefore be different from those which a bar of the metal would yield even if the bar had the same grain-size as the wire. Wires smaller than the one used would undoubtedly give still different values for the modulus of rigidity. The determination of the change of the rigidity with the size of the wire would make very interesting work but the labor involved would be very great.

In conclusion I wish to express my appreciation to Professor L. P. Sieg for his many valuable suggestions and keen interest in this work; and to the Director of the Nela Research Laboratory who, through Dr. A. G. Worthing, so kindly furnished the tungsten and sealing-in wires, the calibrated standard 'amp and the pyrometer lamp. I am also indebted to J. B. Dempster and M. Teeuween for much valuable information regarding workshop methods, and to J. C. Steinberg who generously helped me prepare the platinum-coated mirrors with his evaporation apparatus.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA,
June 6, 1923.[11]

[10] Sieg, Proc. Iowa Acad. Sci. **24**, 207, 1917

[11] This work was completed in July, 1921. Present address is University of Oklahoma, Norman, Oklahoma.

THE AUDITORY MASKING OF ONE PURE TONE BY ANOTHER AND ITS PROBABLE RELATION TO THE DYNAMICS OF THE INNER EAR

By R. L. Wegel and C. E. Lane

Abstract

Auditory masking of one pure tone by another.—Using an air damped telephone receiver supplied with current with a proper combination of two frequencies, as source, the amount of masking by tones of frequency 200 to 3500 was determined for frequencies from 150 to 5000 per sec. The magnitude of a tone is taken as the logarithm of the ratio of its pressure to the threshold value, and masking is taken as the logarithm of its threshold value with masking to that without. The curves of masking as function of magnitude are approximated straight lines as a rule except for rounded feet, of slope s intersecting the magnitude axis at minimum masking magnitude m. For a given masking frequency n the slope increases from zero through nearly 1.0 for a frequency near n, then more slowly, approaching about 3 to 4 for the highest frequencies measured. The intercept is small or zero below n, then increases rapidly, approaching the value 3 for high frequencies. Except when the frequencies are so close together as to produce beats, the masking is greatest for tones nearly alike. When the masking tone is loud it masks tones of higher frequency better than those of frequency lower than itself. When the masking tone is weak, there is little difference. If the masking tone is introduced into the opposite ear, no appreciable masking occurs until the intensity is sufficient to reach the listening ear through the bones of the head. At intensities considerably above minimum audibility, there is no longer a linear relation between the sound pressure and the response of the ear. Data are given showing *combinational tones* resulting from this non-linearity when two tones are simultaneously introduced in the ear. The presence also of *subjective overtones* in a loud tone accounts for the large amount of masking of tones higher than itself by a loud masking tone.

Dynamics of inner ear.—The data on masking together with Knudson's data on frequency sensibility are interpreted in terms of the *dynamical theory of the cochlea* which ascribes its frequency selectivity to a passing of vibrations along the basilar membrane and a shunting through narrow regions of the membrane at points depending on the frequency. Conjectured curves are given for a few single frequencies of the amplitude of vibration of this membrane as a function of the distance along it.

Part I. Auditory Masking of One Pure Tone by Another

1. Introduction

In past work on audition very little attention has been given to the phenomenon of masking. A. M. Mayer[1] has left a more complete record of observations on masking than any one else. He concludes that

[1] Mayer, Phil. Mag. **11**, 500, 1876

low frequency sounds may completely "obliterate" higher frequencies of considerable intensity but higher frequencies do not "obliterate" lower ones. In his experiments he used organ pipes for low frequencies and tuning forks for the high ones. With the organ pipe sounding, the action of the fork could be made intermittent by moving the hand to and fro over the mouth of a resonance box with which it was used. He describes his results in part as follows: "As the vibrations of the fork run down in amplitude, the sensations of its effect become less and less until they soon entirely vanish. Indeed the vibrations of the forks may be suddenly and totally stopped without the ear being able to detect the fact. But if instead of stopping the fork when it becomes inaudible, we stop the sound of the organ-pipe, it is impossible not to feel surprised at the strong sound of the fork which the open pipe had smothered and had rendered powerless to affect the ear. No sound, even when very intense, can diminish or obliterate the sensation of a concurrent sound which is lower in pitch. This was proved by experiments similar to the last, but differing in having the more intense sound higher (instead of lower) in pitch."

The experiments of Mayer are of course only qualitative. The work described in this paper was undertaken to obtain quantitative data and to find an explanation for the phenomena observed.

2. *Definition of masking.* Unless otherwise specified, whenever the term masking is used, it is intended to mean the masking of one tone by another when both are introduced in the same ear. For convenience in presenting data, the magnitude of a tone is defined as the ratio of its pressure to that of its minimum audible value. A logarithmic scale is used in plotting. If a minimum audible pressure of one tone is p_1 and the introduction of a second tone changes its minimum detectable value to p_2, the ratio p_2/p_1 is taken as the magnitude of the masking of the first tone by the second and is likewise plotted on a logarithmic scale. In this paper the term pressure is used to signify the root mean square value of the sound pressure in the external ear passage.

3. *Range of intensities and frequencies of the tones used.* Fig. 1 is a diagram of the auditory sensation area.[2] This figure practically describes itself. It shows that the region of sensation covered by these experiments includes the most important range of frequencies and intensities. The higher levels in this region near the threshold of feeling are generally impracticable for extended experimentation because they induce tinnitus. The very low and very high frequencies are not covered in this work because they are of lesser importance, and furthermore, intense and

[2] See Wegel, Proc. Nat. Acad. Sci., July, 1922 and Bell System Tech. J., Nov. 1922.

pure tones at low frequencies and intense tones at high frequencies are produced with difficulty.

4. *Masking as a function of intensity.* Fig. 2 shows the amount of masking of various frequencies from 250 to 4,000 cycles produced by an 800 cycle tone, plotted as a function of the magnitude of this masking tone. For example, the first curve shows the amount of masking, as already defined, of 250 cycles plotted as a function of the magnitude of an 800 cycle masking tone. Each plotted point represents the average of

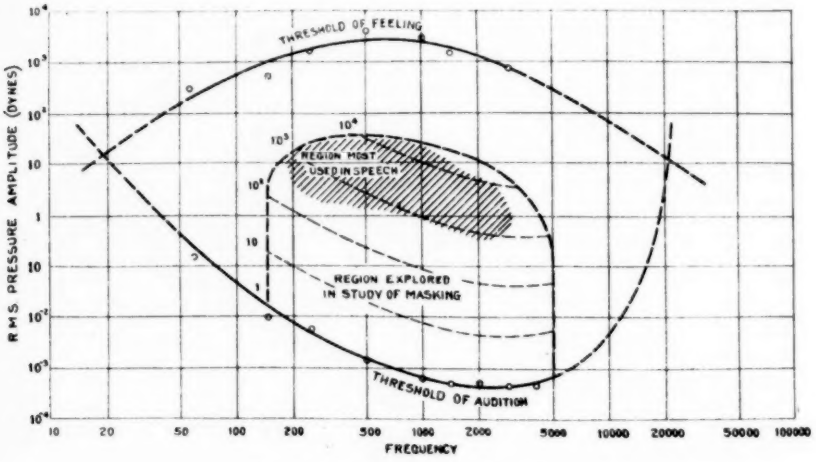


Fig. 1. Auditory sensation area.

four observations taken in succession at one time. In all figures the masking tone is designated by F_1 and the masked by F_2. These curves were all obtained for one observer except the dash-dot ones, which are included here to show the small amount of variation usually found when taking masking data for different observers of normal hearing.

The curves do not all pass through the origin as they evidently should except, as will be explained later, when the tones produce beats. The minimum audible reference value used was the average taken over a period of several days. Plotted in this way it was found that such curves varied from day to day near minimum audibility but for the higher intensities checked within the experimental error. The general magnitude of the deviation of the lower end of the curve from the origin will be seen from the variations shown by the curves. This shows that the ear is quite variable in its behavior near minimum audibility, but comparatively constant for louder tones. The curves as corrected by the dotted lines represent a close approximation, in each case, to the average curve which would have been obtained if the observations had extended over a long

period of time. This correction does not apply when the tones produce beats.

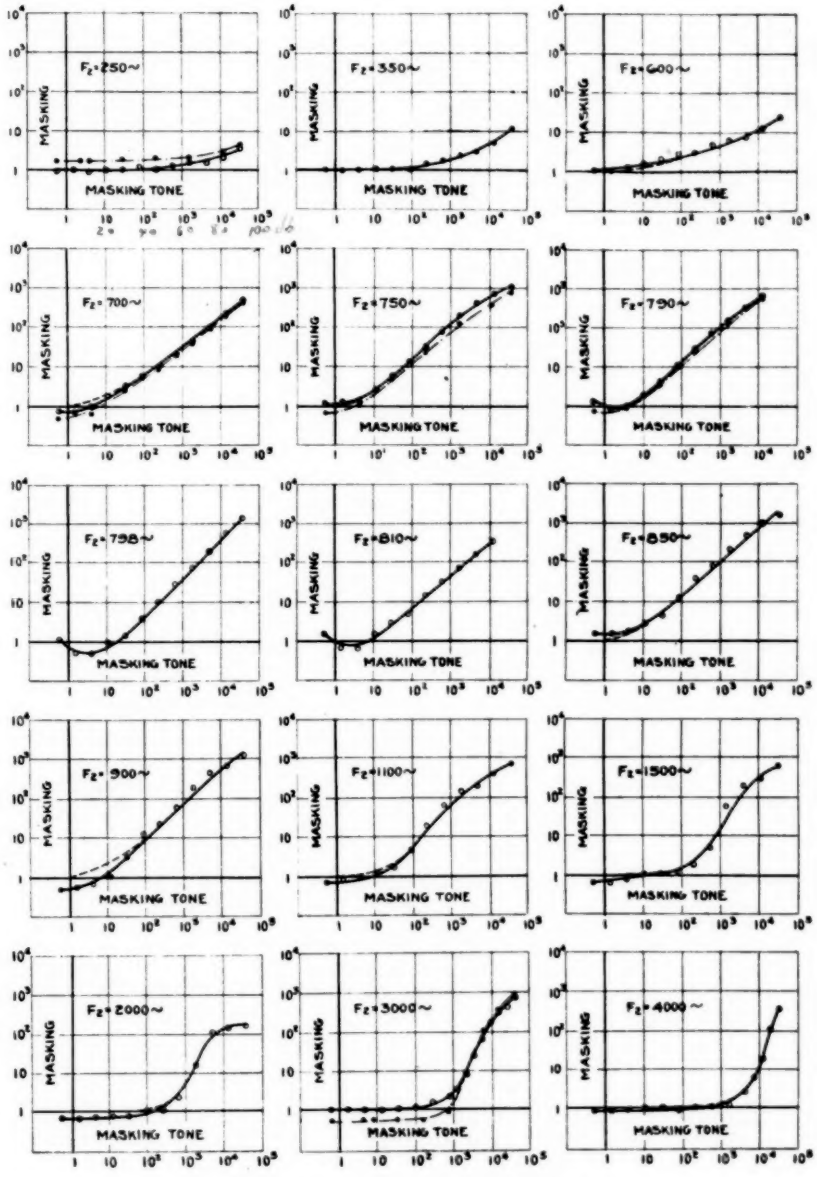


Fig. 2. Data for masking tone of 800 cycles.

Fig. 3 shows some of the corrected curves from Fig. 2 reproduced on common axes. Curves for masking tones of 200, 300, 400, 600, 1200,

1800, 2400 and 3500 cycles are also included in this figure. The frequency of the masked tone is indicated on each curve.

From the figures certain general facts are evident. A tone of a frequency much below the masking tone is not perceptibly masked for the lower range of intensities and hardly more than perceptibly so when the tones are very loud. A tone of much higher frequency than the masking

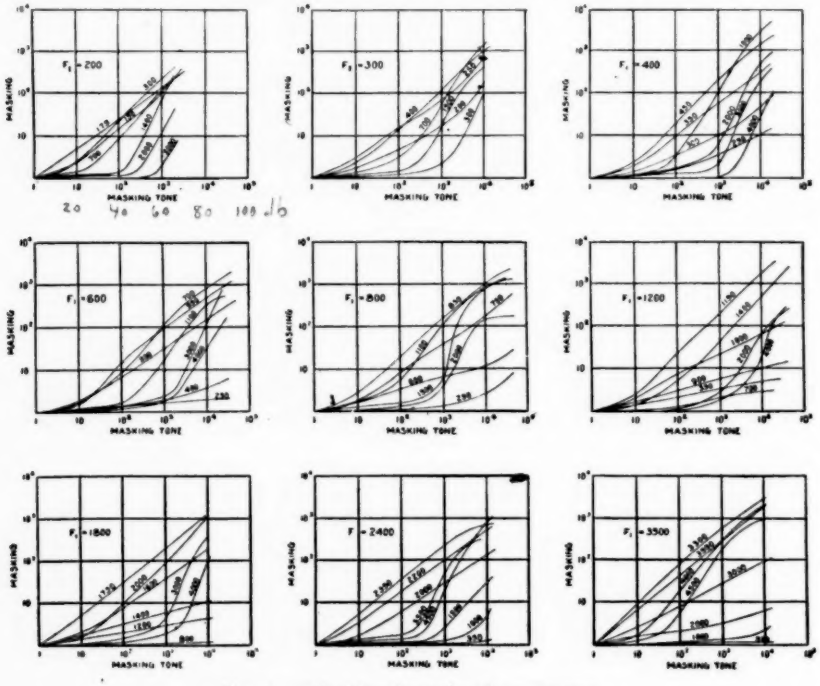


Fig. 3. Masking for tones in same ear.

tone is not perceptibly masked for the lower range of intensities, but at a rather definite high intensity masking occurs perceptibly and quickly becomes very great as the masking tone is increased. In general, masking is greater when the tones lie close together, the curves approaching straight lines with 45° slopes, intercepting the axis of abscissas at about ten times the minimum audible pressure of the masking tone.

When the tones are close enough together in frequency to beat, they do not give masking curves in the same sense as when farther apart. They represent measurements of the minimum perceptible fluctuation of the beating tone. Two such tones, separately inaudible, but each not lower than one-half the minimum audible pressure, will obviously beat when introduced together in such a way as to be alternately audible and inaudible. This effect accounts for the depression in the curve at low

intensities in Fig. 2 for $F_2 = 790$, 789 and 810. At higher intensities the magnitude of the minimum perceptible beating fluctuation may be obtained from the difference between abscissas and ordinates. The minimum detectable amount of this fluctuation has been found to decrease as the beat frequency decreases, approaching a value which would be expected from ear sensibility data.

The sudden increase in slope of the curves when the masked frequency is higher than the masking frequency is associated with the appearance of combinational tones. The curve in Fig. 2 for $F_2 = 2000$ shows a decided bending over at high intensities. This and similar bendings may be accounted for on the supposition that both tones are conducted through the head to the opposite ear in such relative amounts that the masked tone is detected there while it is still masked in the ear to which the sound is applied. It has been found that a small amount of bone conduction takes place between the receiver ear cap and the mastoid bone. The phenomena of combinational tones and head conduction will be discussed later.

5. *Masking as a function of frequency.* Fig. 4 shows the masking of tones of various frequencies by a masking tone of 1200 cycles at 160, 1000 and 10,000 times its minimum audible value. With the exception of

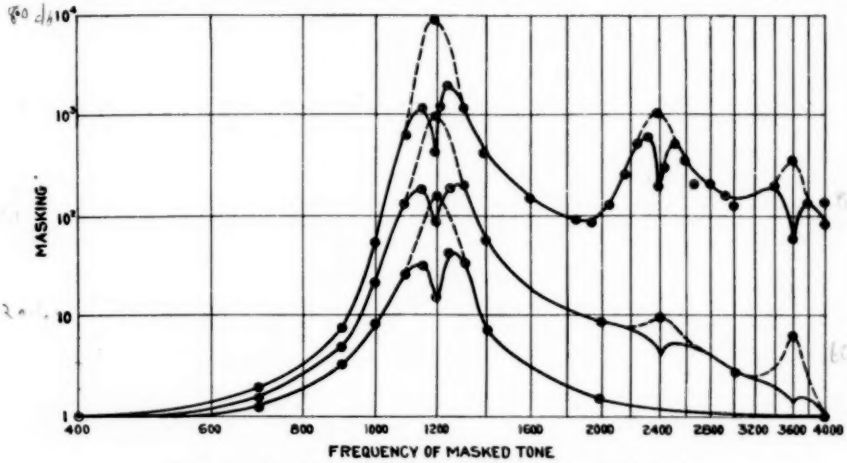


Fig. 4. Masking of various frequencies by 1200 cycles.

the region near the masking frequency where beats occur, the lowest curve shows a gradual falling off of masking as the masked frequency departs on either side from the masking frequency. Within the region of beats, the amount of masking decreases as the masking frequency is approached. The intersection of the curves at the masking frequency may be interpreted to give intensity sensibility. The highest curve differs in that its

characteristics in the region of the first and second overtones of the masking frequency are much like those in the neighborhood of the masking frequency. It resembles such a curve as might be expected from a knowledge of the lowest curve if three masking frequencies, 1200, 2400 and 3600 cycles were present, with relative magnitudes of 1:0.1:0.025. An harmonic analysis of the sound as picked up by a condenser transmitter, showed that these tones were not appreciably present in the air. These and other tests were made, in fact, in all measurements recorded in this paper, and in no case was the distortion in the receiver detectable. Since beats are obtained at the frequencies of overtones, it is concluded that these harmonics are introduced subjectively in the ear due to some non-linear characteristic of its response. The magnitude of these overtones may be obtained experimentally by increasing the intensity of a secondary tone of such a frequency as to beat with the harmonic, to a point where beats are most prominent and taking the intensity of this

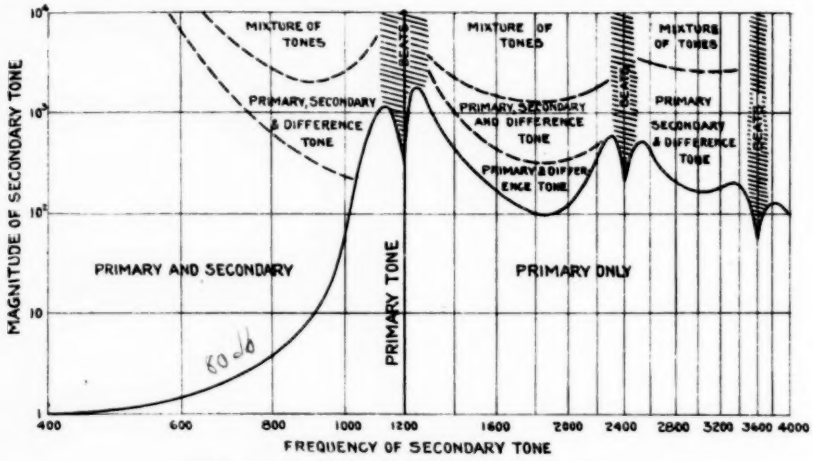


Fig. 5. Sensation caused by two pure tones.

tone to be equal to the intensity of the overtone. The dots represent the magnitudes of the harmonics so determined. This method is not very accurate because the intensity of the variable tone at which the most prominent beats are heard tends to be somewhat higher than the fixed overtone. The middle curve represents a transition between the other two. It indicates harmonic components of relative magnitudes 1:.01:.006. These curves show that a tone masks frequencies higher than itself better than lower frequencies only when it is loud.

6. *Non-linearity of response of the ear.* The character of the sensation, when two tones are acting together on the ear, varies considerably with the relative frequency and intensity values. Fig. 5 represents the sensa-

tion caused by a tone of fixed frequency, 1200 cycles, and magnitude 10^4, in combination with various secondary tones of which the frequencies are represented by the abscissas and the magnitudes by the ordinates. The continuous curve is the same as the top curve in Fig. 11. The various areas represent ranges of magnitude and frequency of the secondary tone in which combinational tones of various kinds, as indicated, appear. As any secondary tone of a frequency below about 1000 cycles, for example 800, is gradually brought up in intensity from a sub-audible value to a point at which it is just detectable, it is first heard as a separate tone along with the primary tone. In the lower part of this range, the intensity of the secondary tone may be increased to very large values and still be perceived independently of the primary. When, however, the intensity of the secondary tone is increased to a point indicated by the dotted line, the difference tone is distinguishable and increases gradually in relative intensity as the area above this line is crossed. At the high intensities in this region, a very complex mixture of tones is heard. When a secondary tone of 1900 cycles is introduced in the same way, its presence is first detected by a difference tone, and the secondary is not heard. As the intensity is further increased, the secondary tone becomes audible along with the difference tone. As the intensity is increased to the higher levels, the mixture of tones becomes more and more complex. With this explanation, the meaning of the rest of the figure will be obvious.

A careful analysis was made of the mixture of tones present in the ear when a primary of 1200 at a magnitude 6×10^4 was present along with a secondary of frequency 700, of about the same intensity. The component frequencies were determined by introducing a third tone of known variable frequency and determining the frequencies at which beats occur. If f_1 represents the primary, and f_2, the secondary, the frequencies found in the mixture were f_1, 1200 cycles; f_2, 700; f_1+f_2, 1900; f_1-f_2, 500; $2f_1$, 2400; $2f_2$, 1400; $3f_1$, 3600; $3f_2$, 2100; $2f_1+f_2$, 3100; $2f_1-f_2$, 1700; $2f_2+f_1$, 2600; $2f_2-f_1$, 200 (?); $4f_2$, 2800; $2f_1+2f_2$, 3800; $2f_1-2f_2$, 1000; $3f_1+f_2$, 4300; $3f_1-f_2$, 2900; $3f_2+f_1$, 3300; $3f_2-f_1$, 900. No attempt was made to determine their magnitudes although this can probably be done approximately by measuring the intensity of the exploring tone at which the beats at each frequency are most prominent. With the exception of the frequency $4f_1$, this series is all that would be expected if the response of the ear were non-linear and represented by the equation:

$$x=a_0+a_1p+a_2p^2+a_3p^3+a_4p^4.$$

In this equation, x is the response of the mechanism of the middle ear; a_0, a_1, a_2, etc., are constants, and p is the pressure in the ear canal. While frequencies introduced by higher powers of the pressure were probably

present, they were very faint and no careful search was made for them. No careful investigation has yet been made of this phase of audition. Results of further work may call for modifications of the interpretation

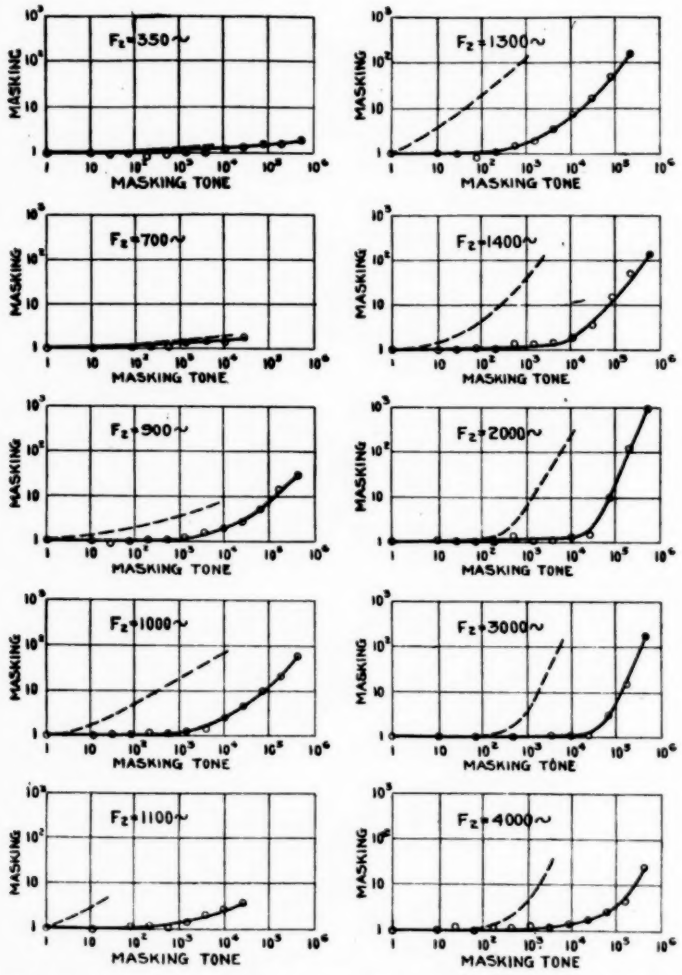


Fig. 6. Masking data for tones in opposite ears, masking tone 1200 cycles.

given here. It may be interesting to note in this connection that one of the striking characteristics of some kinds of abnormal hearing has been found[2] to be an exaggerated departure from linearity.

7. *Masking with tones in opposite ears.* Fig. 6 gives the masking when the masked and masking tones are introduced in opposite ears. The dotted curves show the corresponding data for tones in the same ear. The two sets of curves are nearly alike except for displacement in the

ratio of $1:10^2$ to $1:10^3$ along the horizontal axis. These curves may be explained by assuming that there are two kinds of masking, central and peripheral, the former being generally relatively small and resulting from the conflict of sensations in the brain and the latter originating from overlapping of stimuli in the end organ. Central masking is probably always present to a certain extent whereas peripheral masking can only occur when the two tones excite the same region on the basilar membrane. All large amounts of masking may be attributed to peripheral masking. The similarity, except for the displacement already noted, of the two sets of masking curves indicates that most of the masking for loud tones in opposite ears is peripheral masking, caused by the conduction of the masking tone through the head to the opposite ear with sufficient intensity to cause peripheral masking there. This presumes an attenuation of the tone through the head from one ear to the other of the same order of magnitude as the displacement between the two sets of masking curves.

The magnitude of the masking tone in these experiments was referred to the minimum audible value for the ear into which it was introduced. The magnitude of the masking was referred to the minimum audible value for the opposite ear. It will be seen, therefore, according to the explanation offered above, that the amount of displacement of a curve gives the sum of the conduction loss through the head and the difference in sensitivity of the two ears. If both had been referred to the minimum audible value of the ear receiving the masked tone, the displacement would have given the attenuation through the head. Since the two ears of the observer did not differ greatly in sensitivity, this displacement gives the proper order of magnitude for the attenuation.

There is still further evidence that when a tone is introduced into one ear by a telephone receiver, the opposite ear is also excited but to a lesser degree. Cases of persons very deaf in one ear have been noted for which 10^2 to 10^3 times the current is required for audition with the receiver on the deaf ear over that for the receiver on the good ear. Also, when the sound for the receiver on the deaf ear is audible, it may be greatly enhanced by placing the finger in the good ear, indicating that the sound is not only heard first in the good ear but that it arrives there by bone conduction. Furthermore when two tones of the proper frequencies to beat are introduced in opposite ears the best beats[3] are always heard when one of the tones is over 100 times the amplitude of the other and the relative intensities for hearing these best beats are nearly indepen-

[3] In this connection, see G. W. Stewart, Phys. Rev. **9**, 514, 1917. Stewart's conclusions are somewhat at variance with those arrived at here.

dent of the sensitivity of the ear to which the louder tone is applied. In fact this ear may be entirely deaf, or, if normal, its sensitivity may be lowered by plugging, and best beats will still be heard at the same relative currents through the receivers.

In view of the approximate agreement of all the evidence of head conduction, it seems safe to conclude that this phenomenon actually exists and that it accounts for the resemblance of the two sets of masking curves in Fig. 6. This attenuation through the head, of course, applies only when telephone receivers are used in the ordinary manner as the sound source. When other sources were used, different values of attenuation were found.

Part II. Dynamics of the Inner Ear

8. *Dynamical theory of the cochlea.* A consideration of the anatomy of the cochlea makes it unreasonable to suppose that the individual hair cells, basilar fibers or rods of corti can act as independent resonators, even assuming that dissipational impedance to their motion is small enough to permit of resonance. The motion of each must be greatly affected by the reactions of others due to their dynamical proximity. One element cannot resonate without setting the others in vibration and itself have a complex motion with component frequencies corresponding to all the modes of motion of the complete system which would be obtained by means of its Lagrange discriminant or the equivalent. These frequencies are not generally the resonance frequencies of the vibrating elements themselves. This sort of consideration leads to a treatment of the dynamics of the cochlea as a whole such as that explained by Roaf.[4]

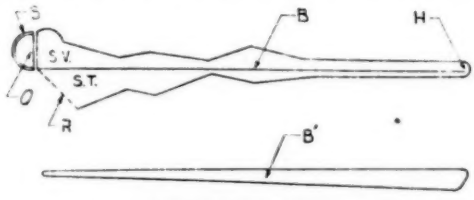
The mechanism assumed here of the action of the cochlea may be explained by reference to Fig. 7a, which represents the cochlea uncoiled.[5] The stapes, in responding to the sound pressure received through the middle ear, is displaced in the oval window, causing a mass movement of the liquid in the scala vestibuli and scala tympani, which except for a small yielding of the labyrinthine walls, results in an equal and opposite displacement of the membrane of the round window. This mass movement of the liquid can take place only by means of the displacement of the basilar membrane or through the helicotrema. If the pressure change is very slow, the movement will take place through the helicotrema. If the pressure change is more rapid, i.e., if the frequency is increased, most of the movement will take place through the displacement of the

[4] Roaf, Phil. Mag., (V) **43**, 49, Feb. 1922

[5] Wrightson, The Analytical Mechanism of the Internal Ear

basilar membrane. This displacement will have a well defined maximum at some definite point, the location of which depends on the frequency, and will decrease rapidly on either side of this maximum. As the frequency is increased, the position of the maximum approaches the proximal end of the membrane.

The motion of the basilar membrane in any region may be assumed to produce a stimulus of the nerve terminals in that region. This stimulus



S = Stapes; O = Fenestra Ovalis; S.V = Scala Vestibuli; S.T. = Scala Tympani; H = Helicotrema; R = Fenestra Rotunda; B' = Surface View of Basilar Membrane.

FIG. 7a. Cochlea uncoiled.

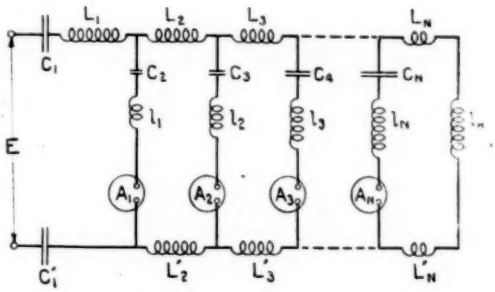


FIG. 7b. Electrical analogue of the cochlea.

may be due to the relative motion of the basilar and tectorial membranes, or flexure of the basilar membrane, or to both. In any case, the amount of the stimulus of any nerve terminal may be taken as a direct function of the motion of the membrane at the point at which it terminates.

A "lumped constant" electrical analogue of the cochlea is shown in Fig. 7b. Although the analogy is not very close, its selective characteristics are similar to those of the cochlea. The inductance L_1 corresponds to the mass of the stapes and its attached parts, C_1, the elasticity restraining its motion, C_1', the elasticity of the round window membrane. The inductances L_2, L_3, etc. represent elements of mass of the fluid in the scala vestibuli, L_2',L_3', etc. similar constants for the scala tympani; C_2, C_3, C_4, etc., the elasticities of elements of the basilar membrane; l_1, l_2, etc. their masses as augmented by contiguous elemental volumes of fluid on either side. A_1, A_2, A_3, etc. are ammeters corresponding to the nerve terminals on the various elements of the membrane. L_h

corresponds to the element of fluid mass in the helicotrema. The series inductances decrease essentially along the structure to correspond with the decreasing cross section of the two scala. The shunt inductances vary more or less in proportion to the widths of successive elements of the membrane. The capacities increase to correspond to the increasing flexibility of the membrane due to its increasing width. Neither the exact magnitudes and variations nor the exact dispositions of the elements of this analogue can be given because the dynamical constants of the parts of the cochlea are not known. Resistances are, of course, associated in various ways with the inductances and capacities. These are not represented in the figure. This electrical network should behave much like the cochlea in that, as the frequency increases, the meter giving a maximum reading is nearer and nearer the source.

9. *Positions on the basilar membrane of maximum response to various frequencies.* It is found that if the two points of a pair of dividers are brought in contact with the back of the hand, the minimum separation at which they can be distinguished separately is about 32 mm. On the finger tips where the nerve terminals are more numerous this distance is about 2.3 mm. According to the theory of the cochlea given above, two frequencies nearly alike cause maximum stimulations at adjacent points on the basilar membrane. The minimum detectable difference in frequency then corresponds to the minimum detectable distance between the corresponding maxima on the membrane. The auditory nerve terminals are quite evenly distributed along the membrane so that it may be assumed as a first approximation that the space interval between two disturbances which are just separately distinguishable is the same all along the membrane. This interval corresponds to the minimum detectable frequency difference, as given by sensibility data, between the tones causing the two disturbances. If f_1 and f_2 are the lower and higher limits to which the basilar membrane responds, l_0 the total length of the membrane, l the distance from the helicotrema to the point at which the disturbance corresponding to a frequency f takes place and Δf the minimum perceptible difference in frequency:

$$l \int_{f_1}^{f_2} \frac{df}{\Delta f} = l_0 \int_{f_1}^{f_2} \frac{df}{\Delta f}$$

The distribution of frequency response along the membrane is plotted in Fig. 8. This was calculated from Knudsen's frequency sensibility data and using the frequency limits 0 and 15000 cycles and 31 mm for the length of the membrane. The sensibility at extremely high frequencies is so small that the value of f_2 may be anything from 10000 cycles to infinity without appreciably affecting the general distribution. Similarly

f_1 may be taken as any frequency from 0 to 50 cycles without materially affecting the distribution because this range is so small compared to the audible range of frequencies. The plot, Fig. 8, shows that 1000 cycles falls about at the middle of the membrane and that distances corresponding to equal frequency intervals decrease rapidly as the stapes end is approached.

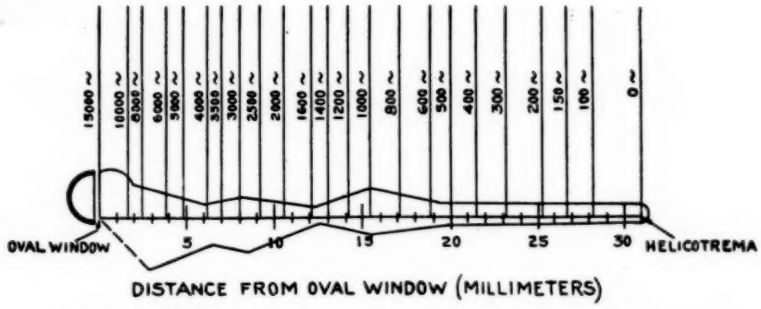


Fig. 8. Characteristic frequency regions on basilar membrane.

It is interesting to compare the space discrimination on the basilar membrane with those on the back of the hand (32mm) and finger tips (2.3 mm). The sensibility data interpreted as above give about .02 mm.

10. *Variation in amplitude of vibration along the basilar membrane for a single frequency.* Fig. 9 shows a hypothetical curve of the variation

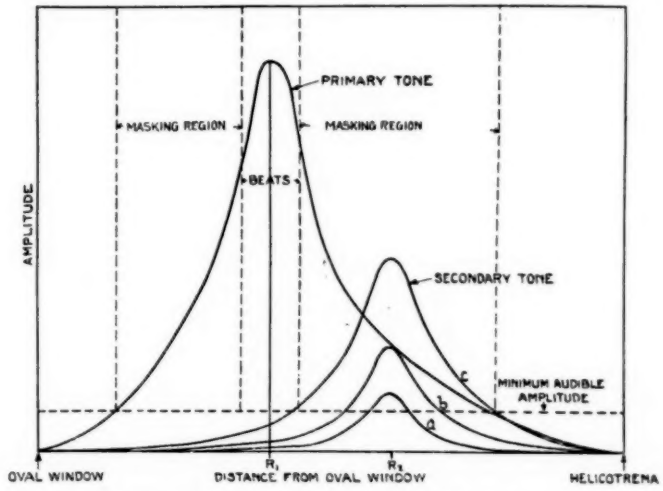


Fig. 9. Hypothetical curve of vibration of basilar membrane.

of the basilar membrane along its length, in response to a primary tone of single frequency. This curve has a maximum at a point in the region R_1, and falls off rapidly on both sides. A dotted line is drawn to show

the minimum amount of vibration necessary to produce perceptible stimulation of the nerve terminals along the length. It may be assumed that the nerves are nearly enough alike, along the membrane, and similarly situated, so that the actual motion necessary to produce minimum audible sensation is about the same at all points. Three other curves corresponding to three different magnitudes of the secondary tone are also shown. These curves have maxima in the region R_2. Curve a corresponds to a stimulus which in the presence of the primary tone is not detectable, but alone is audible. Curve c corresponds to a stimulus which is detectable in the presence of the primary. Some magnitude between curves a and c must correspond to a minimum detectable magnitude of the secondary tone. This is a magnitude at which some definite relation exists between the amplitude caused by the secondary tone in the region R_2, and the amplitude at the same place caused by the primary tone. These amplitudes have been tentatively assumed equal in this work, that is, that the secondary tone, when just detectable, is represented by curve b. The acceptance of a ratio of different order of magnitude is unreasonable and a fine discrimination cannot be justified at this time. Fragmentary evidence, which will not be gone into here, indicates that the ratio should not differ greatly from unity. On this assumption, if central masking is neglected, a secondary tone may be used as an exploring stimulus to measure the amplitude of motion, due to a primary tone, of the basiliar membrane at various points along it except in remote regions where the amplitude is less than that necessary for sensation, and very near the maximum, where the primary and exploring tones beat.

The maximum amplitude on the membrane due to any single tone, in units of the minimum audible amplitude of the membrane, is its "magnitude" as already defined. The maximum value of the curve of a primary tone is therefore given by its magnitude. The amplitude of vibration at any point in regions where masking occurs is given by the magnitude of masking of the exploring or secondary tone.

11. *The curves of vibration of the basilar membrane at different frequencies.* Fig. 10 shows the vibration of the basilar membrane for different frequencies as determined from experimental data by the method described in the preceding section, for amplitudes corresponding to the same pressure of .5 dyne per cm^2 in the external ear canal. This corresponds to a sound which is not so loud as to produce noticeable harmonics. The dotted curve is the locus of the maxima for all frequencies at a constant pressure of .5 dyne in the external ear canal for the ear on which the measurement was made. This curve represents the average of data

taken over a long period of time so that irregularities which are usually present in a single curve are eliminated. The unit is as before the amplitude of the membrane necessary for minimum audibility but the scale is arithmetic. The curves become less sharp as the frequency is decreased. This is in agreement with what would be expected from the dynamical structure of the cochlea. At very low frequencies the stimulus may be conceived of as due to a more or less bodily motion of the tectorial membrane along the basilar membrane.

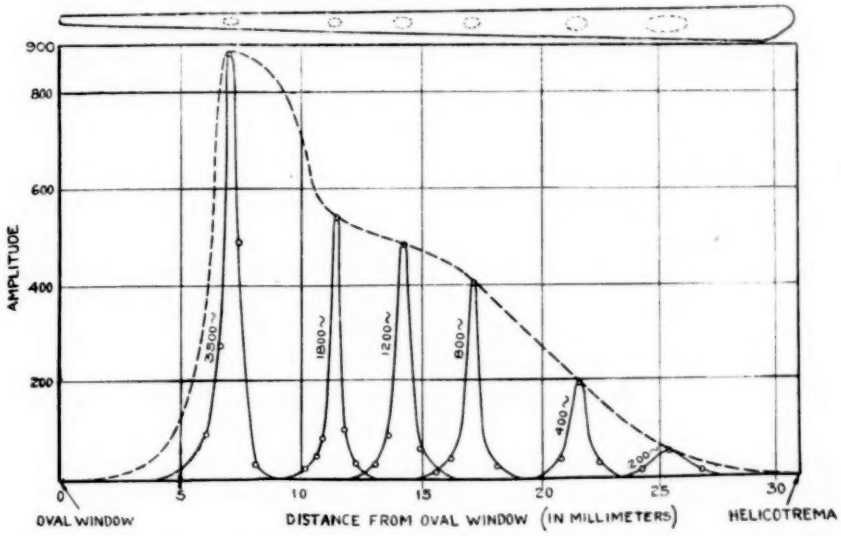


Fig. 10. Amplitude along basilar membrane for different frequencies; r. m. s. pressure 5 dynes.

It is to be expected that similar curves at extremely high frequencies should become less definite, probably not by becoming flatter, but by having their maxima at or beyond the proximal end of the organ of corti. This conclusion is arrived at principally from a consideration of the curves of absolute sensitivity of normal ears (see Fig. 1) in which the sensitivity is seen to drop off very sharply at about 15,000 cycles.[6] This sort of an assumption is further substantiated by the fact that when plotted as displacement of the basilar membrane, sensitivity curves of abnormal ears in which the lesion can be reasonably well traced to degeneration of the nerves of the proximal end of the basilar membrane, also indicate similar sharp cut-offs at frequencies much lower than 15,000 cycles whereas no such abrupt cut-offs of lower frequencies have yet been

[6] For original data see C. E. Lane, Phys. Rev. **19**, 492, May 1922

recorded.[7] According to this theory of the action of the cochlea, it follows that as long as there is sensitivity in any of the nerves, even if it is only in a small region, the ear will be able to detect any frequency if it is loud enough, and that it will detect with greatest sensitivity those frequencies for which the sensitive region is characteristic.

A plan view of the basilar membrane is shown drawn to scale, at the top of the figure. Conjectured contour lines are drawn enclosing areas over which the amplitude is more than one half that of their centers. The lengths of these areas are obtained from the curves shown in the figure and their widths by taking one half the width of the membrane.

Fig. 11 shows curves of response of the basilar membrane for two frequencies, 1200 and 3500, at constant amplitudes of the membrane of 8000 times the minimum audible amplitude. This, of course, did not represent equal pressures in the external ear canal. The secondary maxima caused

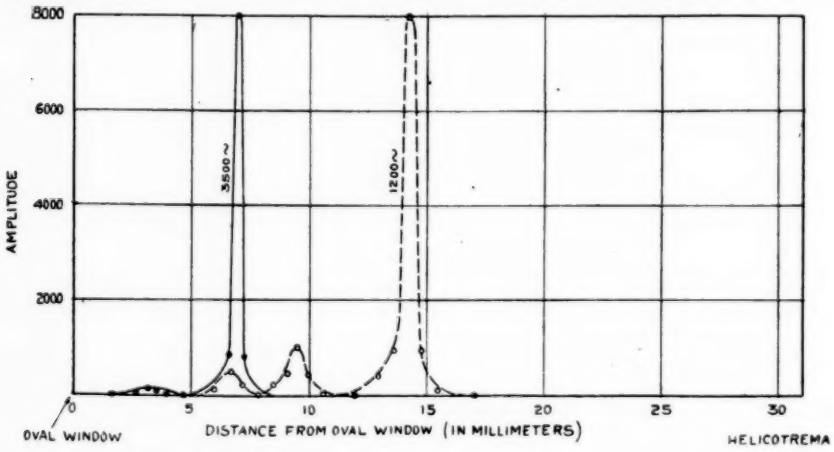


Fig. 11. Amplitude along basilar membrane for loud tones.

by the subjective overtones are present in each case. It seems most reasonable to ascribe the non-linearity producing these overtones to some part of the middle ear, possible the joint between the hammer and anvil which may have enough static friction to give a rubbing effect when vibrating violently.

The conjectured vibration of the membrane with two loud tones of 1200 and 700 cycles under conditions described in section 5 is qualitatively shown in Fig. 12. The vertical lines indicate the positions of maxima. Their magnitudes cannot be given at this time. This indicates that a

[7] For example, see E. P. Fowler and R. L. Wegel, Audiometric Methods and Their Application, Trans. Am. Laryngological, Rhinological and Otological Society, 1922.

large portion of the membrane responds to comparatively simple stimuli when they are loud.

12. *Discussion.* A tentative interpretation of the principal pertinent data available has been made in terms of the theory that the cochlea separates vibrations according to their frequencies, projecting them so to

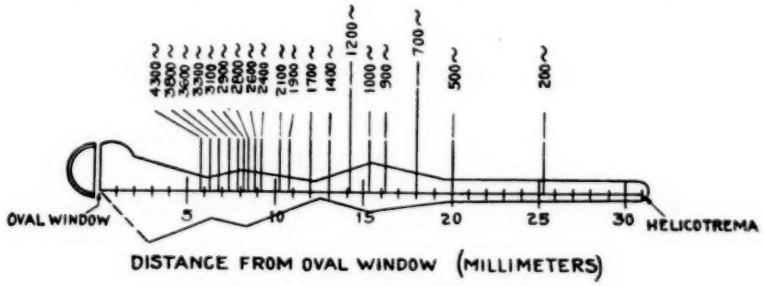


Fig. 12. Combinational tones.

speak, from the stapes through their various appropriate regions of the basilar membrane where they are sensed, and then passing them back to the round window where their pressures are relieved. While this interpretation is probably the simplest that could be made, and the present data seem to be in accord with it, it is of course, possible that other satisfactory interpretations might be found, though they are not very obvious. On account of the inaccessibility of the ear, the determination of its dynamics depends on indirect methods such as those in this paper. Many details as given here might with equal justification have been varied quantitatively to a certain extent. It is hardly worth while going into a further discussion of these details in view of the limited amount of applicable data upon which they depend.

It might be well to explain a little more in detail the general features of the mode of perception of relative pitch and intensities. The brain is assumed to detect differences in pitch simply by experience in associating the stimulation of different groups of nerves with different pitches. Differences in intensity may be detected either by the violence of agitation of nerve terminals at the position of its characteristic maximum or by bringing into play new terminals at the sides of the peak which at lower intensities are subject to subaudible stimulus or more likely both. It is often observed that a small change in intensity is mistaken for a change in frequency. This means that the brain can detect very small changes in position or altitude of the vibration curve of the basilar membrane, but cannot distinguish between these changes unless they exceed a certain definite amount.

The exact position of the maxima on the membrane must vary with intensity because of the varying effect of non-linearity. All curves given in this paper are located on the membrane by means of the sensibility curve at 100 times the minimum audible amplitude. The exact location of these maxima is also quantitatively uncertain as already stated because of a lack of the knowledge of distribution of sensory terminals on the membrane and the relation between this distribution and the minimum detectable distance between maxima. The frequency sensibility and the widths of the peaks of vibration might be expected to be related. The fragmentary data bearing on this point show no simple agreement but the frequency sensibility is the most logical to use at this time in determining the location of frequency response on the membrane.

Nothing has been said of the action of the apparatus of the middle ear. The data given here do not directly bear on that problem. If the minimum audible stimulating motion at all nerve terminals is the same, then the dotted curve of Fig. 10 obtained from minimum audibility gives the frequency characteristics of the combination of middle ear apparatus and cochlea. A determination of the mechanics of this combination or its elements will probably have to be done indirectly from this standpoint.*

Appendix: Apparatus and Method

The apparatus was essentially the same as previously used in this laboratory in the determination of the frequency-sensitivity of normal ears.[8] An air damped telephone receiver was used as the sound source. The currents at different frequencies were supplied by means of special vacuum tube oscillators equipped with filters to eliminate effects due to harmonics. Two voltage attenuators were used, one for the primary, and one for the secondary frequency. These attenuators were of the dial type reading voltage directly on a logarithmic scale and having a total range of 1 to 10^6. In taking data on masking for both tones in the same ear the output of the attenuators was connected in series with the receiver. In this way it was possible to vary the receiver voltage for each frequency independently. The minimum audible voltage for each frequency was separately determined, keeping the other considerably below minimum audibility. The primary voltage was then kept constant at different

* It should be obvious, contrary to an impression apparently created (J. P. Minton, Phys. Rev. **22**, 506, Nov. 1923) in the discussion of this subject at the April 1923 meeting of the Physical Society at Washington, that our assumptions attribute masking ultimately to an inability of the brain to perceive separately, two stimuli on the basilar membrane which are caused by motions of this membrane bearing the relations given in section 10 of this paper.

[8] Fletcher and Wegel, Phys. Rev. **19**, 553, June, 1922

levels above minimum audibility while the secondary was gradually brought up from below minimum audibility until its presence produced a just noticeable change in what was heard while listening to the primary. The ratio of the just detectable voltage of the secondary in the presence of the primary to the minimum audible voltage of the secondary was taken as the corresponding pressure ratio, hence the amount of masking as defined in Part I was found. The ratio of the primary voltage as used to its minimum audible voltage gave its magnitude. In the case of tones in opposite ears the procedure was nearly the same. Two receivers, one for each ear, were used in connection with separate attenuators. The primary tone in one ear was set at a definite magnitude above minimum audibility for that ear, and the amount of masking of the secondary tone in the opposite ear observed.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY
AND WESTERN ELECTRIC COMPANY, INC.,
July 23, 1923.

CORRECTIONS

Scattering of X-Rays by Crystals. G. E. M. Jauncey. Vol. 20, p. 405, November, 1922.—Figures 2 and 6 should be interchanged.

Coefficients of Viscosity and Slip of Carbon Dioxide. James M. Eglin. Vol. 22, p. 161, August, 1923.—On p. 162, l. 16, AT should be ΔT. On p. 164, l. 3, 0.83 should be 0.813.

Magnetic and Natural Rotatory Dispersion in Absorbing Media. E. O. Hulburt. Vol. 22, p. 180, August, 1923.—On p. 181, Eq. (2), hs should be h_s. On p. 182, l. 4 introduce brackets around $1/2\pi$. On p. 183, Eq. (15), h, should be h_1. On p. 186, in Eqs. (26), (27) and (28), write μ_0 instead of μ_0^2.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE CINCINNATI MEETING, DECEMBER 27, 28, AND 29, 1923.

The 25th Annual Meeting (124th regular meeting) of the American Physical Society was held at the University of Cincinnati, Cincinnati, Ohio, on Thursday, Friday, and Saturday, December 27, 28, and 29, 1923. The presiding officer was Professor C. E. Mendenhall, President of the Society. On Thursday afternoon at 2:00 o'clock, Professor Mendenhall delivered the presidential address, his subject being "Some Recent Developments in the Study of the Solid State." Dean Louis T. More presided during this address. On Friday evening, December 28, there was a dinner for members of the Society and friends at the Hotel Sinton, attended by one hundred and thirty-four persons. The average attendance was about two hundred and fifty.

The annual joint session with Section B was held on Friday afternoon, with Professor W. F. G. Swann, Chairman of Section B, presiding. Section C and the American Chemical Society took part in this joint session. The retiring Vice-President, Professor F. A. Saunders, opened the joint meeting with an address entitled "Some Aspects of Modern Spectroscopy." This was followed by a symposium on "The Reflection and Scattering of X-Rays," consisting of three addresses as follows: "The Scattering of X-Rays," by A. H. Compton, University of Chicago; "Scattering from Other Elements than Carbon," by Bergen Davis, Columbia University; "The Scattering and Reflection of Short X-Rays," by William Duane, Harvard University.

On the evening of Friday, December 28, there was a dinner for members of the Society and friends at the Hotel Sinton, attended by one hundred and thirty-four persons.

The regular annual business meeting of the Physical Society was held on Saturday morning, December 29, 1923, at 11:00 o'clock. A canvass of the ballots for officers resulted in the elections for the year 1924 as follows:

For President:	Charles E. Mendenhall
For Vice-President:	Dayton C. Miller
For Secretary:	Harold W. Webb
For Treasurer:	George B. Pegram
For Members of the Council, Four-year term:	A. S. Eve A. W. Hull
For Members of the Board of Editors of the Physical Review, Three-year term:	H. D. Babcock W. G. Cady John T. Tate

The Secretary reported that during the year there had been 152 elections to membership and that the resignations of 31 had been received. The deaths of nine members had been reported during the year.

At the meeting of the Council held on December 27, 1923, three persons were elected to Fellowship, 14 were transferred from Membership to Fellowship and nine elected to Membership. *Elected to Fellowship:* Robert H. Baker, Stanislaw Loria, Francis D. Murnaghan. *Transferred from Membership to Fellowship*: G. Breit, Vannevar Bush, E. A. Eckhardt, John S. Foster, I. C. Gardner, Franklin L. Hunt, Loyd A. Jones, John P. Minton, Chauncey G. Peters, Alexander D. Ross, R. A. Sawyer, Wilmer Souder, A. Q. Tool, Milton S. Van Dusen. *Elected to Membership:* Rolla V. Cook, Duruard L. Eaton, James M. Eglin, John M. Grzybowski, Clive B. Meredith, Henry L. Moore, Walter B. Morehouse, L. M. Mott-Smith, Leon E. Smith.

The regular program of the Physical Society consisted of 79 papers, 12 of which were read by title. The abstracts of these papers are given in the following pages. An **Author Index** will be found at the end.

HAROLD W. WEBB, *Secretary.*

ABSTRACTS OF PAPERS

1. The number of Gamma rays emitted per second from radium B and C in equilibrium with a gram of radium and the number emitted per atom disintegrating. ALOIS F. KOVARIK, Yale University.—The γ rays are considered as entities. When γ rays pass through matter β rays are produced and their ionization phenomena can be counted, thus obtaining a count of the γ rays. Al, Cu, Sn, Pt, Pb, of several thicknesses, were used as fronts of counting chambers. All constants used in the calculation were obtained under experimental conditions. The methods used to determine the constants and the various corrections to be applied are discussed. Two series of experiments were

carried on, one for the total heterogeneous γ radiation and one for hard γ radiation. The ratio of the latter to the former was checked by an ionization method. The possibility of β particles being emitted at different times and different places by the same γ ray is considered and it is concluded to be highly improbable. The total γ rays from radium B and C per second per gram of radium in equilibrium was found to be 7.28×10^{10}, i.e. within 2 per cent of the number of atoms of radium B and radium C disintegrating.

2. The scattering of X-rays. J. A. GRAY, McGill University.—Recent experiments give further proof that when x-rays are scattered their frequency is diminished and that this diminution in frequency is (a) greater the greater the angle of scattering; (b) independent of the thickness and nature of the radiator; (c) approximately independent of the wave-length of the primary rays over the range used. From radiators of high atomic weight a still softer radiation is emitted which is regarded provisionally as being excited by the photo-electrons (see Trans. Roy. Soc. Can. May 1923).

3. Direction of ejection of photo-electrons by polarized x-rays. F. W. BUBB, Washington University.—Stereoscopic photographs were obtained, by Wilson's cloud method, which show the ionized tracks of photo-electrons ejected by plane polarized x-rays. The photographs, taken with the plate at right angles to the beam, show two types of asymmetry in the direction of ejection of the photo-electrons. There is a strong concentration of photo-electrons ejected nearly parallel to the electric vector of the polarized beam. Stereoscopic examination of the photographs shows one-sixth of the photo-electrons are ejected with a component opposite to the direction of the beam, one third ejected approximately at right angles to the beam and one-half ejected with a component in the direction of the beam. Numerous very short tracks whose initial portions are in the forward direction of the beam, may be interpreted as due to the recoil electrons predicted by A. H. Compton. To explain the lateral asymmetry on a corpuscular quantum theory we must assume that the quantum is a vector bundle of energy since it acts as though exploding at right angles to its direction of motion and in a definite plane. (See above p. 137 for complete article.)

4. A quantum theory of the direction of ejection of photo-electrons. F. W. BUBB, Chicago University.—Mackenzie's observation of the longitudinal and the writer's observation of the lateral asymmetry in the direction of ejection of photo-electrons suggest the hypotheses: (a) A quantum of radiation which ejects a photo-electron from an atom imparts its energy $h\nu$ to the atom by exerting a force parallel to the electric vector of the radiation; (b) When a quantum of radiant energy is imparted to an atom to eject from it a photo-electron, the forward momentum $h\nu/c$ of the quantum is delivered to the photo-electron. By taking into account the initial momentum of the electron in its atomic orbit, explanation is given of the lack of exact coincidence in direction of the electric vector and the initial portion of the path of the photo-electron. This also explains why certain photo-electrons are ejected with a component opposite to the beam. The theory leads to the equation, $R = (\lambda + .110\sqrt{\lambda_0})/(\lambda - .110\sqrt{\lambda_0})$, where R is the ratio of the forward to the backward photo-electric current from a thin plate traversed normally by homogeneous radiation of wave-length λ, and where λ_0 is the critical ionization wave-length of the element composing the plate. The results of Bragg and those of Beatty check the equation respectively for variations of λ_0 and λ. Cooksey's results for extremely thin films are in very close agreement numerically. Among other deductions the theory agrees with Beatty's observation that the kinetic energy of the photo-electron is independent of its direction of ejection and it explains qualitatively the writer's distribution curve for photo-electrons ejected by plane polarized x-rays.

5. Angular distribution of recoil electrons produced by polarized x-rays. G. E. M. JAUNCEY, Washington University.—The writer has developed a corpuscular quan-

tum theory of the scattering of both unpolarized and polarized x-rays (Phys. Rev., **22**, 233, 1923; Proc. Am. Phys. Soc. Chicago, 1923). This theory is applied to the number of electrons recoiling in different directions when polarized x-rays are used. The incident quantum $h\nu_0$ strikes the electron with the result that a quantum $h\nu_\phi$ goes off in the direction ϕ while the electron recoils in the direction θ, where $\cot \phi/2 = (1+\alpha) \tan\theta$ and $\alpha = h/mc\lambda_0$. Knowing the distribution of the quanta $h\nu_\phi$ it is possible to calculate the number N_θ of electrons recoiling in a direction θ and crossing one cm² on a sphere of radius R. Formulas containing F are derived where $F=2\alpha$ or $2\alpha+\alpha^2$ (as in the previous paper). For $F=2\alpha+\alpha^2$, N_θ is proportional to $\cos\theta \cos^2 2\theta/(1+\alpha)^2$ when the electrons recoil in the plane containing the electric vector and the primary beam of x-rays. For the perpendicular plane, N_θ is proportional to $\cos\theta/(1+\alpha)^2$.

6. The intensity of x-rays scattered from rocksalt. G. E. M. JAUNCEY and H. L. MAY, Washington University.—X-rays from a Mo target (47.5 kv) were passed through a zirconium phosphate filter and were scattered by a thin slab of rocksalt, and the absolute value of the scattered intensity between 5° and 90° was measured. The writers believe that the excess scattering observed by others for amorphous substances at about 20° to 30° may have been due to a powdered crystal effect. This effect can be eliminated by scattering from a large crystal, since then the Laue spots occur only at particular angles between which there is only diffuse scattering. However rocksalt shows excess scattering; hence true excess scattering exists. The experimental scattering curve corresponds hardly at all (except as regards order of magnitude) with Thomson's theory. Neither does the experimental curve agree with the theory of P. Debye (Ann. der Phys., **43**, 49, 1914). A better agreement however is found with the quantum theories of scattering of Compton and Jauncey at angles greater than 90°. (See p. 128.)

7. Crystal reflection and change of wave-length. P. A. ROSS, Stanford University.—If a frequency shift occurs on reflection from a crystal corresponding to the shift $\Delta\lambda = .048 \sin^2 \frac{1}{2}\theta$ found by Compton for paraffin, aluminium, and graphite, then, if an x-ray beam be reflected from two crystals in succession, the angle of reflection at the second crystal should be slightly different from that at the first. If two spectrographs are placed in series so that the α_1 line from the first passes through the slit of the second, this twice reflected ray may have a different wave-length and fall in a different position on the plate than would a single reflected ray obtained by direct passage from a bulb through the spectrograph. Spectrogams were made of such single and double reflected rays on the same plate. No relative displacements of the lines could be seen in the first, second or third order. No shift can exist amounting to as much as one tenth of the amount that might be expected.

8. The intensities of reflection of the characteristic rays of palladium from fluorite. DUNCAN A. MACINNES and THEODORE SHEDLOVSKY, Massachusetts Institute of Technology.—Using a modified Bragg apparatus, it was possible to measure accurately the relative intensities of the various orders of reflection from three different crystal faces of fluorite. The results, when plotted with intensities as ordinates and a function of the angle of reflection as abscissas, group themselves into three sets of smooth curves: (a) Those resulting from reflection of both the calcium and the fluorine atoms, (b) those from calcium atoms alone, and (c) those resulting from the difference between calcium and fluorine atoms. These results have been obtained with the alpha doublet and with the beta lines of palladium. Attempts to relate these curves to each other have failed, however, due to the presence of abnormal absorption at angles at which reflection takes place.

9. The refraction of x-rays in calcite. C. C. HATLEY and BERGEN DAVIS, Columbia University.—The departure from the law $n\lambda = 2d \sin\theta$ for crystal reflection at different orders has been ascribed by Stenström to refraction in the crystal. This re-

fraction in calcite has been approximately measured by Davis and Terrill. The bending of the Mo Kα rays due to refraction in a split calcite crystal is only 3″ of arc. The refractive effect was increased very much (from 3″ to 30″ of arc) by grinding the surface of the crystal so that when the rays were at normal reflection angle θ to the planes, the angle with the surface was very small, less than one degree. Measurements were made by a specially constructed and calibrated ionization spectrometer. The index of refraction was derived from the observed departures from Bragg's law at 1st and 3rd orders. A second method was used in which the lower half of the crystal surface was a cleavage surface and the upper half was ground to an angle to the planes. By the difference in the positions of the first order reflections for the two cases the index of refraction can be derived directly. The weighted mean of the results gives $\delta = 2 \times 10^{-6}$ where $\delta = 1 - \mu$ and μ is the index of refraction. The Lorentz dispersion formula for the frequency of Mo Kα gives $\delta = 1.9 \times 10^{-6}$.

10. Refraction of x-rays in pyrites. BERGEN DAVIS and ROBERT VON NARDROFF, Columbia University.—A determination of the refraction of x-rays in calcite by the method of cutting the crystal has already been made by Hatley and Davis. The experiments are here extended to pyrites. The use of pyrites as compared with calcite enables one to test the dependence on the material of the Lorentz dispersion formula. By cutting and polishing the crystal so that the surface made an angle to the planes, the bending of the rays on passing through the surface was increased from 3.6″ of arc for the crystal not cut, to 160″ of arc for the cut and polished crystal. This increased bending was found to give a great departure from the law $n\lambda = 2\,d\,\sin\theta$ when measured at 1st and 4th orders. The experiments were made with a special ionization spectrometer having an optically calibrated tangent-worm and hand wheel. Measurements were made for both the Mo Kα ($\lambda = .7078$) and Kβ ($\lambda = .6311$). The weighted means of the experimental results give $\delta = 2.82 \times 10^{-6}$ (β line) and $\delta = 3.33 \times 10^{-6}$ (α line). The corresponding values calculated by the Lorentz dispersion formula are: $\delta = 2.62 \times 10^{-6}$ and 3.29×10^{-6}.

11. The absorption coefficients of homogeneous x-rays between wave-lengths 0.1 A and 0.71 A. S. J. M. ALLEN, University of Cincinnati.—The general radiation from a tungsten Coolidge x-ray tube reflected from a crystal was examined from $\lambda = .10$ to $\lambda = .51$ A for the total mass absorption, in the metals C, Mg, Al, S, Fe, Ni, Zn, Ag, Sn, W, Pt, Au, Pb, and Bi. The α and β radiation from a Mo tube was also examined for $\lambda = .63$ A and .71 A. Systematic calculations showed that, if properly chosen values of the scattering were chosen, the results could be expressed by formulas of the form $\mu/\rho = C\lambda^n N^p + \sigma/\rho$ over the range of λ and for N up to .74. The empirical values which seemed to give best results were: $n = 2.92$, $p = 2.92$, $C_K = 7.82 \times 10^{-3}$, $C_L = 1.0 \times 10^{-3}$, $\sigma/\rho = .144$ for Al to .70 for Bi, at $\lambda = .12$ A with increasing values as λ increased. The heavy elements do not agree with the above formula, the "jump" in μ/ρ from L to K absorption decreasing from 5.60 for W to 3.30 for Bi, whilst for Ag it was 7.85. In the atomic form, $C_K = 2.19 \times 10^{-26}$, and $p = 4$. Very good agreement was found with the results of Richtmyer and Duane, wherever they paralleled.

12. X-ray absorption coefficients in the neighborhood of K-limits. F. W. WARBURTON and F. K. RICHTMYER, Cornell University.—Atomic absorption coefficients on both sides of the respective K-limits, have been determined for Mo, Ag, Sn, W, Au and Pb. (1) Assuming Mosley's law $\sqrt{\mu} \propto (N - b)$ and the previously determined law of x-ray absorption $\mu_a = kN^4\lambda^3 + \sigma_a$, the value of $\tau_a (= kN^4\lambda^3)$ just at the short wave-length edge of λ_K should be proportional to $N_4/(N-b)^6$. This is verified experimentally if $b = 7$. (2) The change in τ_a on passing the K-limit is in the ratio of about 7 to 1, but varies with N, decreasing as N becomes large. (3) The ratio τ_K/μ_a decreases as N increases, showing that for elements of higher atomic number, a smaller proportion of the absorbed energy

is radiated as K fluorescent radiation. (4) On the long wave-length side of the K-limit we find, as previously reported, $\mu_a = k'N^2\lambda^3 + \mu_a$ where x has the value 4.3.

13. The relative number of K and L electrons expelled by x-rays. F. K. RICHTMYER, Cornell University.—The change in the absorption coefficient at the K-limit gives the relative amounts of energy extracted from the incident x-ray beams by virtue of the photo-electric emission of K and of (L+M) electrons. This ratio is of the same order of magnitude as, but somewhat smaller than the ratio of the energies required to remove a single K and a single L electron from an atom of the absorber. Assuming that these ratios are identical, one can compute from known data the part of the (L+M) absorption which is due to the L electrons only. The ratio of K absorption to L absorption then turns out to be very nearly the same as the ratio of the K ionization potential to the L ionization potential. Both ratios decrease with increasing atomic number. From this it is concluded that the probability of the ejection of a K electron bears to the probability of the ejection of an L electron a ratio independent of the relative numbers of those electrons and inversely proportional to the respective ionization potentials. If Compton's theory of scattering were true, however, this ratio apparently should be in part dependent on the relative number of K and of L electrons.

14. X-ray crystal analysis of materials in their natural state, with an improved spectrometer. WALTER SOLLER, University of Cincinnati (Introduced by S. J. M. Allen).—A Bragg x-ray spectrometer was equipped with two specially designed collimators, in place of the ordinary slits, each consisting of a large number of parallel strips of metal arranged in a vertical plane, one being placed between the specimen and the target, the other between the specimen and the ionization chamber. The target was placed so that its plane was perpendicular to the axis of the collimator. The first collimator allows only an essentially parallel beam of large cross-section to irradiate the specimen, the second analyses only those pencils of rays which are almost parallel to a given direction. Experiments showed that the α radiation from a Mo water-cooled tube could be resolved into the α_1 and α_2 components when reflected from a specimen of commercial iron strip. When the strip was heated through 450°C, the observed shift of the peaks corresponded to the shift to be expected from calculation of the thermal expansion of the crystals. When the strip was stretched mechanically, there was observed a decided shift of the peaks. With this spectrometer it is not necessary in any way to prepare the specimen by etching or powdering.

15. Precision measurements of the lattice constants of pure metals. WHEELER P. DAVEY, Research Laboratory, General Electric Co.—In certain work in metallurgy it is desirable to know the lattice constants of pure metals with considerable accuracy. Precision measurements have therefore been made of most of the metals in common use, using the purest specimens we have been able to obtain. Results are tabulated below.

Metal	Purity	Structure	Lattice Constant	Density (X-ray data)	Density (Literature)
Al	99.97%†	F. C. C.	4.046±.004A	2.688±.008	2.708
Ni	99.55	F. C. C.	3.499±.003	9.036±.027	8.6 − 8.93
Fe	"C. P."	B. C. C.	2.861±.003	7.864±.023	7.85 − 7.88
Cn	99.98	F. C. C.	3.598±.004	9.002±.027	8.937
Ag	"C. P."	F. C. C.	4.058±.004	10.64 ±.03	10.503−10.53
Mo	98	B. C. C.	3.136±.003	10.265±.030	10.28
Pt	*	F. C. C.	3.913±.004	21.490±.063	21.16 −21.49
Au	25 caret	F. C. C.	4.076±.004	19.210±.057	19.27 −19.32
Pb	99.96	F. C. C.	4.920±.005	11.477±.033	11.341−11.347
Bi	**	Rhombohedral	a=4.539±.005, c/a=2.606	9.796	9.781− 9.799
W	99.995	B. C. C.	3.155±.003	19.317±.057	18.77 −19.3
	98.58	B. C. C.	3.158±.003	19.26	

† From Dr. J. E. Edwards, Aluminum Co. of America. * Market purity; contains Ir in small quantity. ** Single crystal grown by Dr. S. Hoyt.

16. A vacuum tube detector of Hertzian waves. G. BREIT, University of Minnesota.—A three electrode receiving tube (type WD11) was tried as a receiver of Hertzian waves of 60 cm wave-length and was found to be more sensitive as a detector of the waves than a crystal detector and galvanometer. The plate and grid are connected together and both are connected to the filament through a galvanometer.

17. The Stark effect in helium which corresponds to that observed in the Balmer series. JOHN STUART FOSTER, National Research Fellow, Yale University.—Previous research has shown that the Stark effect patterns associated with the diffuse and "diffuse principal" series in both systems of the helium spectrum present an appearance similar to that for the Balmer series, but have fewer components. As the Balmer series is a series of triplets with new lines appearing in very weak electric fields, so in helium there are the corresponding sharp, principal, and diffuse series with combination lines appearing in stronger fields. In hydrogen the two groups of fine structure lines are very close together; in helium the lines of either group are more scattered, and the two groups have a great separation. Thus the Stark effects for the series represented by the formulas, $1P-mQ$, and $1S-mQ$, where $m=2, 3, 4 \ldots$ and $Q=S, P, D, F, G, H$, *etc.* need to be combined to supply in parhelium what corresponds to the Stark effect in the Balmer series. The effect in the doublet system is quite analogous. It will be noted that the formulas include both groups previously compared with the Balmer series. This view is supported in detail by recent photographs which show many new lines.

18. New combination series in the helium spectrum. JOHN STUART FOSTER, National Research Fellow, Yale University.—Near each diffuse line of parhelium there appears, in weak electric fields, a group of lines the number of which increases with the order. The well-known series $1P-mP$ is formed by selecting from each group the line on the extreme violet side. Lines next to the D lines are now found to fit the series $1P-mF$, and the groups are completed by series which may be designated as $1P-mG$, $1P-mH$, etc. Direct measurements of these lines from $m=4$ to $m=6$ have been made at very low fields. In only one case does a wave-length thus observed differ from the calculated value by more than 0.2 A. In higher fields a similar group of lines is found near each member of Stark's "diffuse principal" series $1S-mD$ of which they have been considered the components. In reality they belong to the series $1S-mD$, $1S-mF$, $1S-mG$, etc., and they may be recognized only by their Stark effects. According to the theory, the effect for each line should resemble that for the corresponding line in the group first considered. Fortunately, the accurate knowledge of the effect of an electric field on the first group as supplied by recent photographs enables the writer to identify with certainty the series in the second group.

19. Resonance radiation from cadmium vapor. A. D. POWER, University of Minnesota.—It was found that if cadmium vapor enclosed in an evacuated quartz tube was illuminated by a spark discharge, the wave-lengths λ3262 ($1S-2p_2$) and λ2289 ($1S-2P$) were radiated strongly by the freshly formed vapor. The first doublet of the subordinate series of thallium λ5350 ($2p_1-2s$) and λ3776 ($2p_2-2s$) also appeared in the radiation from the fresh cadmium vapor, although the cadmium spark did not reveal the presence of this impurity.

20. Spectroscopic evidence of impact ionization by positive ions in Mercury vapor. JOHN T. TATE, University of Minnesota.—*Positive sodium ions* from a heated platinum strip coated with sodium phosphate were accelerated into an ionization chamber containing mercury vapor. There was no electric field in the chamber hence delta ray electrons from the walls would have their initial velocities only. Both sodium and mercury arc lines appeared in the chamber when the velocity of the positive ions exceeded 40 volts. While there was no definite evidence that this is a critical ionizing potential

for positive ions it is believed that this is the best evidence yet obtained of impact ionization by positive ions at low velocities. When a field much in excess of 15 volts was set up in the chamber the character and position of the glow changed, indicating that the delta ray electrons were beginning to ionize the vapor. The mercury lines were very much more prominent under these circumstances. *Positive mercury ions* produced by electronic collisions in mercury vapor were accelerated into the ionization chamber. The arc lines of mercury appeared when the velocity of the ions exceeded 70 volts.

21. The excitation function of the mercury spectrum. JOHN A. ELDRIDGE, University of Wisconsin.—The relationship between the line intensity and the exciting voltage is such as to show that at least for low pressures and currents, ionization plays an unimportant role in spectrum excitation. A striking difference exists between lines which originate in electrons falling from an S, D or d_2 orbit and those in which the electron came from an s, d_1 or d_3 orbit. The first class of lines get stronger and the second diminish with increasing energy of the primary electrons. To get the electron into the orbits of the first class requires a larger energy of the impacting electron, because, probably, a re-orientation of the plane of the orbit in space must be effected.

22. The metastable state in mercury vapor. HAROLD W. WEBB and LUCY J. HAYNER, Columbia University.—Using a four-electrode tube and the alternating current method described before (Phys. Rev. **21**, 479 and 791, 1923) the persistence of λ 2537 excited in mercury vapor by electron impacts has been further studied. This persistence was found to be independent of the temperature (from 25° to 70°C), but to depend markedly upon the tube dimensions. These results show that a $2p$ metastable state is produced by the electron impacts and that this persistence is not due to "imprisonment of radiation," i.e. repeated absorption and remission. Assuming the radiation to be carried to the photo-electric surface by the metastable atoms, curves were calculated agreeing very closely with the experimental curves, showing that the diffusion of the excited atoms is the primary cause of the diffusion of the radiation. Further evidence is given by photographs of a quartz flask containing mercury vapor and an electrode system. When λ 2537 is excited near the center, the central grid can be seen, contrary to the "imprisonment" theory which requires a very small average path for the radiation. Quartz cells containing mercury vapor excited by an external intermittent arc also show a persistence of λ 2537, increasing linearly with the length of the cell.

23. Behavior of mercury arc lines after removal of the exciting potential. LUCY J. HAYNER, Columbia University.—By means of a rotating commutator carrying on its shaft a disk with a small hole serving as the slit of a quartz spectrograph, the persistence of the mercury arc spectrum was studied after the exciting potential had been cut off. In all cases λ 2537 was especially strong in the after spectrum, its rate of decay being nearly independent of the vapor pressure, but increasing with decreasing size of tube. At all temperatures (40°C−200°C) λ 2537 also exhibited a second type of persistence, weaker and of much longer duration than the first, and increasing in intensity with the temperature. The arc lines were extinguished upon removal of the voltage, but reappeared after an interval which apparently depended upon the rate of decay of λ 2537. Principal series lines were not found in this after spectrum. The lines $2p_2-1s$, $2p_i-md_j$, and $2p_i-mD$ were particularly strong. These persistent lines were not affected by reverse voltages sufficient to prevent recombination. There is evidence that the extinction of the arc lines immediately after removal of the potential is due to absorption by excited atoms in the $2p$ state as observed by Kurth (Phys. Rev. **22**, 202,1923).

24. The band spectrum of nitrogen, and its theoretical interpretation. RAYMOND T. BIRGE, University of California.—A preliminary investigation, with the following conclusions: (1) A common final quantum state of the second and fourth band groups is also the initial state of the first group. Hence the *excitation potential* of the second

group should be about four volts higher than that of the first, in agreement with experiment. (2) The mathematical relationships of these three groups indicate only one possible assignment of true *vibration quantum numbers*. This assignment leads immediately to a possible explanation of various changes in physical appearance, i.e. (a) of various bands, particularly λ3371 (the 0→0 band) with self-induction (Lewis), (b) of various bands with slow electron impact (Meggers, Foote, and Mohler), (c) in bromine mixtures (Datta), and (d) with temperature variations (Angerer, Kirschbaum, etc.). (3) *Active nitrogen* is apparently a metastable energized neutral N_2 molecule, chiefly with vibration quantum number eleven. (4) All three series of the 3371 "triplet" band have been traced through their turning-points, and their points of zero intensity definitely located at $m=8$, thus establishing the quantum structure of "triplet" bands. (5) Two independent methods have been developed for obtaining the *moment of inertia for the N_2 neutral molecule* in the non-vibratory initial state of the second group, the results agreeing to within one per cent, and giving 17.2×10^{-40}. (6) The identification, on internal evidence, of the CN bands with N_2 is shown to be false.

25. The excitation of the spectra of nitrogen by electron impacts. O. S. DUFFENDACK, University of Michigan, and D. C. DUNCAN, Pennsylvania State College.—The excitation of the spectra of nitrogen was investigated by two methods: (1) In low voltage arcs in a mixture of nitrogen and mercury vapor below the ionizing potential and in nitrogen alone above the ionizing potential; (2) In a three element tube at low pressures. The *first and second positive bands* appear below the ionizing potential and are assigned to the neutral molecule. Two new low potential groups were found in the ultra-violet, one degraded toward the violet and one toward the red. The latter behave like the second positive group and are probably physically related to them. The third positive bands did not appear and are probably due to a nitrogen oxide. The cyanogen bands were likewise absent. The *negative bands* appear above the ionizing potential and increase with the degree of ionization, being emitted by the ionized molecule. A new group of bands in the ultra violet degraded toward the red appears simultaneously and exhibits a similar behavior, indicating a physical relationship. The *line spectrum* appeared above about 70 volts in ordinary arcs and as low as 30 volts in intense arcs, in agreement with the change in the rate of clean-up of nitrogen in the discharge.

26. Use of half quantum numbers in interpretation of hydrogen chloride absorption bands. WALTER F. COLBY, University of Michigan.—A faint band has been observed for hydrogen chloride at 300°C which may be attributed to a vibrational quantum change from 1→2. When computed, however, from the well known bands 0→1 and 0→2 with the usual numbering, the calculated band is displaced relative to the observed band by about one-fourth of the distance between lines. If one assigns to the electronic angular momentum a value of $\frac{1}{2}h/2\pi$, one accounts not only for the missing centers of the old bands, but also for the measured lines of the new faint combination band.

27. A theory of the continuous spectrum. R. D. KLEEMAN, Union College.—This theory is based on the fact that the mass of an atom is located mainly at its nucleus, and that its electrical moment M is given by (J. Franklin Inst., p. 479, Oct. 1923) $M=10^{-19}N^{3/5}$ where N denotes its atomic number. It can easily be shown that if the atom is placed in an electric field of intensity E, measured in e.s.u., it will oscillate about the nucleus with a period T given by $T=6.01\times10^{-4}xN^{1/6}E^{-1/2}$ where x denotes the radius of gyration of the electronic mass of the atom. The value of x in the case of the hydrogen atom is $.53\times10^{-8}$cm according to Bohr's theory of line spectra, and in the case of other atoms is obviously of the order of 10^{-8}cm. The field E acting on an atom in a substance would be produced by adjacent free electrons and other atoms, whose motions would give rise to a variation of E in intensity and direction. Calculation shows that continuous radiation would in this manner be produced ranging in frequency from the longest heat waves to the shortest waves in the ultra-violet region, and that

the intensity of the radiation, and the proportion of radiation of higher to that of lower frequency increases with the temperature.

28. A spectroscopic study of the cathodo-luminescence of fluorite. FRANCES G. WICK, Vassar College.—The luminescence spectra of some specimens of fluorite under cathode-ray excitation consist of broad, diffuse bands in the visible and ultra-violet upon which are superimposed narrow, line-like bands. Visual and photographic study of these line-like bands verifies conclusions of Urbain that they are due to rare earth impurities. Observations are difficult because of low intensity of narrow bands. Heating specimens to 700° C or less before bombardment intensifies narrow bands and weakens diffuse bands. Fusion of fluorite in an oxy-hydrogen flame and subsequent heating in an ordinary blast produces oxidation and brings out a much brighter luminescence spectrum very rich in sharp lines most prominent in the red end of the spectrum. Comparison of position and relative intensities of these lines with lines observed by Urbain in cathodo-luminescence of calcium oxide containing small quantities of rare earths shows very good agreement. The conclusion is reached that the lines are due to rare earths the most conspicuous of which are samarium, dysprosium, europium, gadolinium, and terbium. The method of oxidation used makes lines exceptionally bright and sharp, facilitating observations. A study of the effect of the velocity of the cathode rays upon the spectra of these oxides is now in progress.

29. Measurements of thermo-phosphorescence of glass produced by radium radiation. R. E. NYSWANDER, University of Denver, and S. C. LIND, Bureau of Mines, Washington, D. C.—Intensities of light emitted by glass colored by exposure to radium radiation and then heated to different temperatures were measured by means of a polarization photometer. The two specimens of violet glass used had been uniformly radiated by radon, one within a year and the other about three years ago. Each specimen was divided for the purpose of making independent observations. The total quantities of light emitted by equal surfaces of the same sample were equal in value and independent of the time rate at which the glass was heated. Glass more recently exposed to the radon emitted the greater quantity of light. A piece of each sample of glass was heated very rapidly to 130°C and then held at this temperature in an electric furnace while the light intensity was measured at time intervals until exhausted. Plotting light intensities as ordinates and time intervals as abscissas, two curves, a total radiation and a constant temperature curve, are obtained. Measurements taken from these curves with a planimeter fit the integrated velocity equation for unimolecular chemical reactions: $k=(1/t)\,[\log a/(a-x)]$ where a is the total quantity of light emitted and x is the quantity emitted during time interval t. The constancy of k does not mean that the reaction taking place is necessarily a chemical one, though this would be a plausible tentative assumption. Values of the constant k were determined for each specimen of glass over a wide range of time intervals.

30. Pressure between cathode and anode in discharge tube. ROBERT J. PIERSOL, Research Laboratory, Westinghouse Elec. and Mfg. Co.—In a vertical discharge tube the upper electrode is suspended in the form of a Jolly balance sensitive to a pressure of 0.04 dynes per cm^2. The electrostatic attraction between the electrodes is 0.05 dynes per cm^2 at a difference of potential of 30,000 volts. In air, hydrogen, and argon at pressures varying from 10^{-2} cm to 10^{-6} cm and with differences of potential from 10,000 to 30,000 volts the pressure of the electric wind is less than the electrostatic pressure. Therefore in the absence of thermionic or photoelectric emission the pressure between the electrodes appears negligible.

31. The minimum velocity of impact to produce secondary electron emission from tungsten. OTTO STUHLMAN, JR., University of North Carolina.—A three element

vacuum tube with tungsten filament, grid and plate was used. Secondary electrons were obtained from the tungsten grid through bombardment by means of thermions accelerated by the potential gradient between grid and filament. To the negative end of the filament was attached a variable negative potential, in series with a galvanometer and the grid, the latter being positive with respect to the filament. A high potential was connected with its positive end to the plate and negative end to the positive end of the grid battery. This kept the plate potential always at a constant positive potential higher than the grid. If secondary electrons were emitted they passed from the grid to the plate and were detected as a decrease in the deflections of the galvanometer. Hence the secondary emission was indicated by a break in the potential-current curve. By connecting the breaks for various filament currents by a straight line an intercept on the voltage axis was obtained which was interpreted as the potential at which secondary emission set in for electrons leaving the filament with zero initial velocity. For tungsten this minimum potential was found to be 3.72 volts.

32. The ionization of hydrogen by electron impact. H. D. SMYTH, Princeton University.—Minor modifications in apparatus and procedure previously used in studying the ionization of nitrogen are described and results of experiments on hydrogen presented. These lead to the conclusion that dissociation does not accompany ionization of molecular hydrogen unless the speed of the impacting electrons is at least 4.2 volts higher than the minimum of about 16 volts required to produce molecular ions. In two series of runs at high pressures atomic ions became the dominant product, suggesting that molecular ions may be dissociated by secondary collisions with other molecules. The bearing of these results on previous work is discussed.

33. The electrical conductivity of active nitrogen. S. KARRER, C. S. FAZEL and B. V. CASSEN, Fixed Nitrogen Research Laboratory, Washington, D. C.—Under certain conditions when nitrogen is passed through an electrical discharge, it is stimulated to emit radiation for some time after it has left the region of the discharge. The electrical conductivity of this radiating nitrogen has been investigated. The relation between the applied electromotive force and current through the gas exhibits saturation effects which depend upon the quantity of active nitrogen and the pressure and purity of the gas. The results indicate the presence of volume and surface ionization effects.

34. On the mobility in air of ions produced in carbon dioxide. HENRY A. ERIKSON, University of Minnesota.—A thin sheet of CO_2 which contained ions produced by rays from polonium was passed parallel to and between two plates between which a current of air of equal velocity was also flowing. The velocity of the air was 2000 cm/sec. The distance from the CO_2 layer through the air stream to the lower plate was 4 cm. By means of a field at right angles to the stream, the positive or negative ions were drawn from the CO_2 layer through the air to the lower plate where the current due to the ions coming to a narrow insulated strip in the lower plate, was measured by means of a quadrant electrometer. By varying the field, the current spectrum was obtained, which is graphically shown by plotting the current as ordinates, and the voltages as abscissas. In the case of air ions, a curve having a single maximum for the negative ion is obtained, whereas in the case of the positive air ion, there is one maximum corresponding to the negative maximum, which gives a positive ion mobility of 1.87 cm/sec/volt/cm, and another maximum at a higher voltage, which corresponds to an ion of mobility 1.36. In the case of CO_2, it is found that there is an *initial positive ion and a final positive ion.* These correspond to the initial and final positive ions in air insofar as they have the same mobility ratio. Each, however, requires 500 additional volts to bring them down as compared with the initial and final air ions, which require 3400 and 4500 volts respectively. It is not as yet clear if the additional 500 volts are required to draw the ions out of the CO_2 stream. In the case of the negative ion in CO_2, it is found that there is an

initial negative ion and a final negative ion. The final negative ion in CO_2 is brought down by 500 volts in addition to the 3400 volts required to bring down the negative ion, or the initial positive ion in air. The initial negative ion is brought down at 3600 volts or 200 volts higher than the air negative ion.

35. The incident and emergent velocities of photo-electrons emitted from thin platinum films. CHIEN CHA, University of Minnesota.—A reinvestigation has been made of the want of symmetry found by various observers for the photo-electric velocity in the cases of incident and emergent light. The film was deposited upon a quartz plate. A beam of light was focussed on the plate through a quartz window, and the transmitted light could be reflected back at will by means of a mirror. In this way it was possible, with minimum change of conditions, to superpose upon the effects produced by the light going one way those produced by its going in the opposite direction. A further variation was possible by turning the film through 180°. The photo-electric velocities were measured by applying a stopping potential to a cylinder surrounding the film. Experiments were first performed with films *evaporated* from a platinum wire, using the unresolved light of the mercury arc and also monochromatic light of wave-lengths 2537, 2150, and 1849 A. No observable change was found on using the mirror. A check upon the observations is the fact that they gave a reasonably good value, 6.41×10^{-27} for Planck's constant h. With films obtained by *cathodic deposition*, however, the mirror had a distinct effect giving, in agreement with former observers, emergent velocities about 20 per cent higher than the incident velocities.

36. Photo-electric emission from platinum as affected by heat treatment. ALBERT E. WOODRUFF, University of Chicago.—The photo-electric emission of platinum as produced by wave-lengths from a mercury-in-quartz arc is caused to disappear entirely by baking out the tube and heating the platinum specimen with an electric current. The cause of the disappearance is proved to be a shift in the long wave-length limit below that transmitted by quartz. This point was tested by the radiation of an incandescent tungsten filament placed inside the tube. When no emission occurred due to the mercury arc a photo-electric current of the order of magnitude to be expected from calculation could be produced by the tungsten filament. Thermionic measurements made on specimens giving no photo-electric emission for the radiation of the mercury arc give values of h well within the values reported by other observers. The observations show that the long wave-length limit of platinum is not constant, and independent of surface conditions, but can be shifted at will.

37. A new photo-electric cell. J. TYKOCINSKI-TYKOCINER and J. KUNZ, University of Illinois.—The photo-electric cell consists of a glass tube supplied with a bulb at each end. Each bulb has inside a hemispherical deposit of metallic potassium and a platinum wire connected with the sensitive layer. Care was taken that the connecting tube itself separating the bulbs had no visible deposit on its wall. When the bulb connected to the negative side of a battery is illuminated, it shows the usual photo-electric effect. If however the transparent connecting tube is also illuminated, the deflection of the galvanometer increases considerably. The illumination of the positively charged anode has no perceptible influence in either of the mentioned cases. If the positively charged potassium layer had the property of reflecting the electrons as observed by Langmuir for caesium on nickel electrodes (Science, November 16, 1923), a decrease of the photo-electric current should be expected when the anode is illuminated. No such action of the positively charged potassium layer was observed. The relation between the photo-electric current and the voltage applied was observed for the case when only the cathode was illuminated and also when both the cathode and the connecting tube were illuminated. The increase of current due to the latter, reaches a maximum for a voltage corresponding to the middle part of the saturation curve. For report of previous investigation see Phys. Rev. **21**, 210, 1922.

38. The variation of thermionic emission with temperature and the concentration of the free electrons within conductors. A. T. WATERMAN, Yale University.—A revised expression for the thermionic saturation current per unit area of the form $i = AT^{\alpha}e^{-\beta/T}$ was derived by the author (Phys. Rev. **22**, 268, 1923, Eq. 9) according to Richardson's thermodynamical method, using (1) the boundary condition that, when ψ the free electron evaporation constant is zero, the free electron concentration inside and outside the hot body is the same; and (2) the interior free electron concentration given by the author's equilibrium theory (l. c. p. 262, Eq. 4) with correction made for an error at valences other than unity. On evaluation, A is found to depend upon the nature of the body, while α depends upon the valence but always lies between the usual ½ and 2. β calculated from the computed A and experimental data shows striking agreement with β observed for Pt, Mo, W, Ni, Ta, Ca, CaO, BaO and SrO. Taking these results as confirmation of the underlying theory, approximate calculation shows the equilibrium concentration of free electrons in a number of metals at 0°C to be of the order of magnitude of 10^{17} per cc. While this order of magnitude appears surprisingly low, yet the usual difficulty with the contribution of free electrons to specific heats of conductors is avoided.

39. The relation between thermionic emission and contact difference of potential. C. DAVISSON, Research Laboratories of the American Telephone and Telegraph Co. and the Western Electric Co., Inc.—If I_1 and I_2 are the densities of thermionic emission from materials (1) and (2) at temperature T, and V is the contact potential of (2) relative to (1) at the same temperature, it follows from considerations first pointed out by Richardson that $\log I_2/I_1 = Ve/kT$. From the same considerations, if plates of materials (1) and (2) at the same temperature are placed in turn opposite a parallel collector, currents of the same density will flow to the collector against a given applied retarding potential, provided this retarding potential is sufficiently great to eliminate the potential minimum caused by space charge. If filaments (1) and (2) of equal diameter and the same temperature are placed in turn in a cylindrical collector, the currents flowing to the cylinder against a given applied retarding potential will be very nearly the same, and their ratio will approach unity as the retarding potential is increased. This relation has been verified, within limits, in experiments on filaments of thoriated tungsten. In a particular test the saturation emission from a filament increased during activation by a factor 1000 while the current to the cylinder against an applied retarding potential of 1 volt increased only by a factor 1.5.

40. The effect of magnetic field on the electronic emission of vacuum tubes. IWAO FUKUSHIMA, University of Minnesota (Introduced by W. F. G. Swann). Assuming Van der Bijl's parabolic equation for the plate current of an amplifying tube $I_p = \alpha(E\rho/\mu + E_g + \epsilon)^2$ the change in the constants α, μ and ϵ were determined for various magnetic field intensities. Both α and μ increased when the magnetic field was in the same direction as that of the electric field. For the detector tube the variations in $\delta^2 I_p/\delta E_g^2$, which is a measure of the detecting power, were determined by obtaining the equations of the characteristic curves with and without the magnetic field. On differentiating these equations it was found that $\delta^2 I_p/\delta E_g^2$ increased with E_g when the magnetic field was perpendicular to the field while without the magnetic field $\delta^2 I_p/\delta E_g^2$ was a maximum when $E_g = 0$. For the experimental verification of the above an alternating e.m.f. of 5×10^{-4} volts and frequency 1.27×10^6 was impressed upon the grid, and using the "null method," an increase in the plate current was noted when the magnetic field was present.

41. The characteristics of pliotrons containing screen-grids. A. W. HULL and N. H. WILLIAMS, Research Laboratory, General Electric Co.—These tests were made on specially constructed tubes containing, in addition to the filament, grid, and plate of the ordinary pliotron, an extra grid between the control grid and the plate. This

extra grid is so placed as to thoroughly shield the control grid and its supporting wires from the plate. In operation it is maintained at a constant positive potential. Its function is to screen the control grid from the electrostatic influence of the plate, thus giving an infinite plate resistance without sacrifice of mutual conductance between grid and plate. This prevents feed-back through the tube from plate to grid circuit. The voltage amplification obtainable under these conditions is the product of mutual conductance by load impedance and is therefore limited only by the load impedance obtainable. The absence of feed-back makes it possible to use highly tuned circuits at any frequency and to connect in series as many stages of amplification as desired without instability or danger of oscillations. Under favorable circumstances, the amplification per stage thus obtained is as high as sixty-fold and by using several stages, amplifications of 15000 have been measured. This result and others still higher that were beyond the range of the measuring device have been obtained without any sign of local oscillations.

42. A singular case of electron tube oscillations. G. BREIT, University of Minnesota.—An oscillation with peculiar properties is described. The wave-length of the oscillation is about 60 cm; it depends critically on the filament temperature and occurs when the plate of the tube is disconnected while a positive potential is applied to the grid. The following additional observations have been made: (a) The potential of the plate becomes highly negative; (b) the oscillation is stimulated by the sudden withdrawal of the magnetic field; (c) the intensity of oscillation is for a certain range dependent and for a certain range independent of whether the temperature is being increased or decreased; (d) the dependence of the oscillation on temperature is changed if the filament current is reversed; (e) the relation between the negative plate voltage and the grid current shows in general marked discontinuities; (f) a magnetic field has a marked influence and is capable of suppressing the oscillation. The present data support an explanation of the oscillation as a space charge effect.

43. Theory of oscillating electric circuit. VINCENT PAGLIARULO, University of Chicago.—Experiments have been performed and a theory is suggested for the operation of the circuit studied by Barkhauser and Kurz (Phys. Zeits. **21**, 1, 1920), consisting of resistance, inductance and capacity, with a positive potential impressed on the plate of the tube. A circuit of this kind differs from an ordinary electron tube radio transmitter in that no arrangement is provided to vary periodically the potential of the grid, which is a necessary condition for the ordinary radio transmitter. Contrary to the view of the above authors that the oscillations are to be accounted for by peculiarities of the motion of the electrons through the grid, a conclusion which has been criticised by others, the author finds that the oscillations can be accounted for by supposing that the resistance of the tube is negative and equal to the metallic resistance of the circuit at the potential at which oscillations occur. This conclusion has been verified by standard measurements on the tube. The order of magnitude of the frequency has been calculated from the constants suggested by the theory and found to be in substantial agreement with the observed frequency.

44. Electrostriction. J. P. KARBLER, University of Cincinnati (introduced by R. C. Gowdy).—According to Maxwell's theory a dielectric placed in an electrostatic field is subjected to a tension along the lines of force and a pressure perpendicular to them. Each of these stresses is given by the expression of $KV^2/8\pi$ per unit area. The present experiments were undertaken in an attempt to measure the mechanical strain in a dielectric along the lines of force, and are a repetition, with more sensitive apparatus, of previous experiments performed in 1904. A specially constructed multi-plate condenser was employed in which the mechanical attraction between successive plates was compensated. The dielectric was composed of rosin, shellac, and Venice turpentine and was cast in vacuum to insure uniformity. The measuring device, which was essen-

tially the "ultra-micrometer" described by Professor Whiddington (Phil. Mag. **40**, 634, 1920), was capable of detecting a change in the length of the condenser of 10^{-8} cm. The condenser was charged to potentials sufficient to produce, according to Maxwell's formula, deformations several hundred times the limit of sensitivity of the ultra-micrometer, but in no case was a contraction of the dielectric observed.

45. A calorimetric method for measuring power factor of insulating materials at radio frequencies. J. E. SHRADER, Drexel Institute.—To measure the power factor of insulating materials at comparatively high voltages where considerable power was lost, a calorimetric method was devised. The source of power was a pair of 250-watt vacuum tubes whose maximum plate voltage was 4000 volts. Water was circulated through coils immersed in mercury which formed the electrodes applied to the sample. The difference of temperature of the water as it entered and left the calorimeter was measured by thermometers. The voltage applied to the sample was determined by an electrostatic voltmeter already described (Shrader, J. Opt. Soc. **6**, 273, 1922). The frequency was regulated by comparison with a Kolster wave meter. For each voltage equilibrium conditions were obtained and the rate of flow of water was observed and the energy loss in heat was computed. The energy loss divided by the input gave the power factor directly. Observations with a number of materials showed no variation of power factor with voltage over a range of 800 to 2300 volts when no appreciable change of temperature took place. The calorimetric method was checked by a substitution method which gave substantially the same results. This work was done while the author was employed by the Westinghouse Electric and Mfg. Co.

46. The variation of metallic conductivity with electrostatic charge. F. WENNER, Bureau of Standards.—This subject has received the consideration of several investigators but whether or not the effect is of sufficient magnitude for definite measurement seems to be an open question, especially in view of the recent work by Perkins. In the present investigation drawn copper wires were used as the conductors and their resistance when charged positively compared with their resistance when charged negatively. However, no change in resistance which could be associated with a change in the sign of the charge was observed. For these comparisons the method employed obviated the difficulties which ordinarily would have been encountered because of changes in resistance resulting from changes in the temperature. Therefore even though the conductors were not in the form of thin films nor the surface charge as large as may have been used previously, the overall sensitivity was exceptionally high. Further, the general behavior of the apparatus was such as to give confidence in its indications. All factors considered, it seems that the effect should have been detected even if smaller by two orders than that which could have been detected in any of the previous work.

47. Note on the electrical conductivity of metals at high frequencies. W. F. G. SWANN, University of Chicago.—The ratio of the conductivity σ_ν for a frequency $\nu/2\pi$ to the conductivity σ_0 for zero frequency has been calculated by various investigators under different assumptions. One of the best known of these calculations is that due to J. J. Thomson, who finds that $\sigma_\nu/\sigma_0 = \sin^2\nu T/\nu^2T^2$, where $2T$ is the interval between two collisions. The expression corresponds approximately to $\sigma_\nu/\sigma_0 = 1-\nu^2T^2/3$. A scrutiny of the assumptions involved in the calculation reveals a certain misuse of the mean free path analogous to that which is occasionally to be found in other problems of the kinetic theory. Since the difference between σ_ν/σ_0 and unity is very small, it results that the error in question is responsible for an error amounting to a factor of twelve in the term $\nu^2T^2/3$. The value for σ_ν/σ_0 when corrected for this error becomes the same as that found by H. A. Wilson by a calculation founded on entirely different bases, and in its applications to the determination of the number of

electrons per cc is less favorable to the free electron hypothesis than is the original formula of J. J. Thomson.

48. Some effects of gas upon the resistance of sputtered platinum films. F. W. REYNOLDS, Cornell University.—Films were deposited on glass, mica, and quartz at various temperatures in residual air at a pressure of 20 microns. Reproducible resistance-temperature data could be taken in vacua below the initial sputtering or ageing temperature. Films deposited on cold surfaces, less than 40°C, suddenly decreased in resistance when O_2 was admitted. H_2 produced very little effect under the same conditions. The resistance of films heated or sputtered between 50° and 350°C always increased when exposed to O_2. The resistance of very thin films may become infinite if they are exposed to O_2 while hot. Rates of increase in resistance as large as 2,500 ohms per minute were measured. This phenomenon is reversible provided the temperature is lowered before an infinite resistance is reached. The resistance-temperature coefficients of these thin films which initially were negative become positive after gas treatment. Although H_2 will cause a large increase in resistance under these conditions, the change is not completely reversible and the resistance-temperature coefficient remains negative.

49. The resistance change of mercury in a transverse magnetic field and the Hall effect in molten bismuth. W. M. NIELSEN, University of Minnesota.—An attempt was made to determine the change in resistance of mercury when placed in a transverse magnetic field. A capillary tube filled with mercury was placed in one arm of a Wheatstone bridge. Preliminary results gave for $\delta r/r$ a value of the order 2×10^{-4}, in agreement with the measurements of Patterson. To eliminate spurious effects caused by the mechanical motion of the mercury, a very fine capillary was finally used; and a pressure of 10000 gm/cm^2 was applied at the two ends of the capillary. No change in the resistance was detected in field strengths from 0 to 16000 gauss, indicating that the ratio $\delta r/r$ for mercury is less than 3×10^{-6}. The work of Eldridge suggests importance of the crystalline structure of metals in the study of the Hall effect. Mercury is known to have a small Hall coefficient. P. Drude and Nernst concluded that the Hall coefficient of molten bismuth was less than 1/60 of the value at room temperature. In view of the fact that most metals have Hall coefficients less than this, work was undertaken to redetermine its value. Direct evidence of the existence of a Hall effect in molten bismuth was not observed. If present it must be less than 2.5×10^{-4} c.q.s. units.

50. An improved form of bifilar electrometer. S. J. MAUCHLY and H. F. JOHNSTON, Department of Terrestrial Magnetism, Carnegie Institution of Washington.—This paper describes a modified form of the bifilar quartz-fiber electrometer devised by Wulf in which various improvements are combined with the advantages inherent in the earlier construction. The chief advantage of the new form is its practical freedom from changes in sensitivity under varying conditions of temperature and humidity. This feature is due chiefly to elimination of the hard-rubber mounting customarily used for supporting the tension-bow and the insulated "inner case" and the substitution of a supporting base which is nearly all metal and, therefore, not subject to humidity effects. Other features provide for improved accessibility to and adjustment of the fiber system. In its present form the bifilar electrometer is admirably adapted to work in electrostatics where it is desired to obtain, by means of a portable instrument, measurements of a precision and accuracy comparable to that obtainable on ordinary circuits by the use of a good voltmeter; it is also adapted to measurements on any electrical system where the requirements include low capacity, negligible period and self-inductance, high insulation, low (usually negligible) temperature coefficient, and freedom from humidity effects.

51. Improved apparatus for recording the electric potential of the air. S. J. MAUCHLY, Department of Terrestrial Magnetism, Carnegie Institution of Washington.

—In the registration of the electric potential of the air frequent tests are made of the essential insulators of the recording system. However, it is seldom practical to make such tests more than once daily, which obviously leaves much to be desired regarding the state of the insulation during the remainder of the day. The apparatus described in this paper was designed to provide the maximum practical safeguard against defective insulation. Automatic insulation tests are made possible by the introduction of an auxiliary sulfur insulator as a section of the rod which supports the collector. This insulator is normally bridged across by a conductor, but once every three hours this contact is broken by a clock-controlled electromagnet. This condition is maintained long enough to allow the registration, successively, of records indicating the approximate state of the essential insulators, the base line, and the approximate state of the auxiliary insulator. From these data it is possible to reject all records which do not conform to an adopted standard of insulation, and thus to arrive at a better knowledge of the diurnal and annual variations of the Earth's electric field than can be obtained without such controls.

52. Cosmic effects in terrestrial magnetism and atmospheric electricity, and their physical bearings. LOUIS A. BAUER, Department of Terrestrial Magnetism, Carnegie Institution of Washington.—As is well known the potential gradient of atmospheric electricity passes through changes of 50 per cent, or more, of its mean value during the day, and from month to month during the year—the so-called diurnal and annual variations. These two sets of variations, besides showing certain local characteristics, exhibit marked phenomena partaking of a world-wide character. There are thus indications that corresponding variations in atmospheric electricity may occur at practically the same absolute time over the entire earth, just as is the case with regard to notable disturbances of the Earth's magnetism. The question here particularly investigated is whether there are also changes in atmospheric electricity from year to year and for successive sun-spot cycles. It is found that there are definite cyclic changes, of world extent, having an average range of about 25 per cent, which are apparently to be associated chiefly with changes in solar activity. This is a matter of considerable importance in the formulation of a theory for the origin and maintenance of the Earth's electric charge. Other physical bearings of the various phenomena disclosed are pointed out.

53. Scope of work at the observatories of the department of terrestrial magnetism of the Carnegie Institution of Washington. J. A. FLEMING, Carnegie Institution of Washington.—Because of the unsatisfactory distribution of magnetic observatories, particularly in the southern hemisphere, and the general lack of provision for the study of atmospheric electricity, earth currents, and like geophysical and cosmical phenomena involved in the problems in terrestrial magnetism, the Carnegie Institution of Washington has authorized the establishment of magnetic and electric observatories where complete programs of work in these subjects may be carried out. Two such completely equipped observatories have been erected, one at Watheroo, Western Australia, and the other at Huancayo, Peru, and there is also an electric observatory in Washington, D. C. In addition to these permanent observatory stations, temporary observatory stations for special studies are established as opportunity offers. The building and instrumental equipment, the scope of automatic records obtained, and detailed particulars are given for the different classes of work enumerated above, together with a brief account of the desiderata essential in the selection of sites and the conditions for operation.

54. Earth currents due to asymmetric heating of the earth by solar radiation. RICHARD HAMER, University of Wisconsin.—Benedicts claims to have discovered the

existence of thermo-electric effects due to asymmetric heating of homogeneous substances. The Earth as a whole due to rotation is evidently subject to asymmetric heating by solar radiation, and will experience earth currents if this effect exists.. These will be most marked in the equatorial regions. The electron flow would be greatest there if these regions were insulated. The actual distribution of land and water of many different electrical conductivities will change the direction and magnitude of these probable earth currents. Moreover, the varying temperature gradients radially, latitudinally and longitudinally, influenced by varying nature of soils, sea-water concentrations and diurnal rotation may possibly give rise to similar electron displacements or earth currents. The resultant earth current for any position will evidently vary with local and general variations of solar radiation. The magnetic effects of these currents would vary accordingly. This theory seems an obvious explanation of careful observations of Dr. Bauer and others on earth current and magnetic variations. Experiments to prove the existence of these earth currents are being carried on. Indications at present seem to support the theory. It is of fundamental importance for it may account for most if not all of the earth's magnetic field.

55. On the present status of the high altitude rocket. R. H. GODDARD, Clark University.—The earlier work on the high altitude rocket research involving the use of smokeless powder was supported by the Smithsonian Institution. Since June 1921, the research has been assisted by two grants from Clark University. It has been possible by actual experiment to produce continuous propulsive force and good combustion, and to prevent serious heating, using liquid combustibles and oxygen, as suggested nine and one-half years ago by the writer and recently again by H. Oberth of Roumania. This method promises simplicity and effectiveness and also the use of hydrogen and oxygen for the greatest altitudes as first suggested by the writer (Smithsonian Misc. Coll. vol. 72). This investigation promises important applications.

56. On determination of magnetometer constants. J. A. FLEMING, Department of Terrestrial Magnetism, Carnegie Institution of Washington.—This paper discusses in detail the methods of determining the constants required for magnetometers using the magnetic method to observe the intensity of the Earth's magnetic field, and the corrections of such instruments on standards in magnetic declination, inclination, and horizontal intensity. So called "absolute constants" may be determined with certain precision by physical measurements, but it is an open question as to whether dependence may not as well be placed upon constants determined relatively only by comparison observations with standard instruments at base stations. Possible changes with time in the values of the constants are of interest. The discussion of numerous determinations of the absolute constants for many C. I. W. magnetometers shows that with appropriate control of a high degree of constancy may be expected except for changes of the moment of inertia of the oscillating magnet and its suspension arrangements. Redeterminations of the moment of inertia after long field trips show surprising agreement between observed inertia changes and observed changes in the corrections of the instruments on adopted standards of intensity. The sources of inaccuracy in ordinary field work are discussed in detail.

57. The hardness of steel and nickel as related to their magnetic properties. S. R. WILLIAMS, Oberlin College.—Eleven different samples of the same piece of nickel were given various degrees of hardness by cold rolling. The values of the scleroscope hardness numbers were then plotted against the changes in length due to a given magnetic field. The same hardness numbers were also compared with the values of the intensity of magnetization of the various nickel rods at a definite field strength. Similar studies were also made of a series of steel rods. In steel the various degrees of

hardness were attained by heat treatment, in nickel the different degrees of hardness were secured by cold rolling. Nickel, as usual, gave some very interesting and surprising results. It is hoped that out of this study a better understanding may come of what is meant by the term hardness.

58. Magnetic hysteresis loops in permalloy. L. W. McKeehan and P. P. Cioffi, Research Laboratories of the American Telephone and Telegraph Co. and Western Electric Co., Inc.—Both symmetrical and unsymmetrical loops have been studied, the former being obtained in these magnetically very soft materials only when stray magnetic fields are carefully compensated. The initial state of magnetization affects both the shape and the mean slope of loops having the same width, even if very narrow. The results obtained furnish, accordingly, criteria for the efficiency of demagnetizing processes, and emphasize the difficulty to be expected in wiping out the previous history in harder magnetic materials.

59. The effect of the material composing the sides of deep slits on the intrinsic intensity of light transmitted by the slits. L. P. Sieg and C. R. Smith, University of Iowa.—Previous experiments (L. P. Sieg and A. T. Fant, J. Opt. Soc. **5**, 218, 1921) have been performed in this laboratory on the intrinsic intensity of light transmitted through deep slits between steel jaws, as a function of the width and depth of the slits, and of the wave-length of the light. The present experiments cover the effects resulting from employing other metals for the jaws of the slits, viz., nickel, gold, copper, and silver. In addition, for the sake of comparison, the results for steel were re-determined. Curves were obtained connecting, for each metal used, intrinsic intensity of the transmitted light as a function of the width of the slits (with the depth a constant) for three well-separated wave-lengths in the visible spectrum. While the curves show the same general form as do those previously obtained for steel, they vary distinctly from one another for different metals under conditions otherwise identical in every respect. Of the materials used to form the walls of the deep slits, nickel, as a rule, permitted the greatest and silver the least intrinsic intensity of light to pass, for all wave-lengths, and for all intermediate widths of the slits. For extremely narrow and for wide slits, the intensity is independent of the material constituting the slit walls. The experimental facts are satisfactorily accounted for by assuming diffraction, coupled with multiple reflections.

60. Improved radiometer construction. J. D. Tear, Nela Research Laboratory.—A fifteenfold increase in the sensitivity of the Nichols radiometer was obtained by attaching "pearled" mica shields immediately behind the usual blackened vanes and by choosing the optimum dimensions and working conditions. In general the sensitivity per unit area increased as the dimensions of the system were reduced, although for vanes smaller than $1.5 \times .5$ mm the sensitivity per unit area was found to be independent of vane width. The proportionality between deflections and radiation intensity was tested by means of a series of rotating sector disks and found to hold accurately. The most efficient fiber is one which gives a half period equal to the desired deflection time and the best gas pressure that which gives critical damping. Thin platinum films are good absorbers of very long infra-red waves and short electric waves. The film thickness for maximum absorption is a critical function of the wave-length.

61. The optical constants of caesium. J. B. Nathanson, Carnegie Institute of Technology.—The caesium mirror was prepared as follows. An intimate mixture of caesium chloride and calcium filings was heated in a vacuum to about 500°C, and the resulting caesium was distilled and finally collected in a small glass cell containing two crown glass windows. The optical constants were calculated by Drude's formulas from the observed values of the phase difference and azimuth of the elliptically polarized

light reflected from the mirror. The method of observation was essentially that employed previously in the determination of the optical constants of rubidium (Phys. Rev. **11**, 227, 1918). The index of refraction of caesium in contact with crown glass diminished from a value of 0.25 at 4550 A to a minimum value of 0.21 at 6000 A, increasing again to 0.23 at 6800 A. The coefficient of absorption was found to increase from 2.3 at 4550 A to 4.1 at 6800 A. The computed reflecting power for normal incidence increased from 0.47 at 4550 A to 0.61 at 6800 A. The values of the reflecting powers so computed were checked by a direct method using a Lummer-Brodhun spectrophotometer. The reflecting powers so determined were found to increase from 0.39 at 4550 A to 0.62 at 6800 A.

62. The optical constants of crystals of selenium and tellurium for wave-lengths from 3000 to 5000 A. R. F. MILLER, University of Iowa (introduced by L. P. Sieg).—The measurements were made by a photographic method, for two positions of the crystal, (1) with the optic axis parallel, and (2) with the axis perpendicular, to the plane of incidence. Two sets of optical constants were found for each substance. For selenium, in the parallel position the index of refraction was found to vary from 3.4 to 4.4, and the reflecting power from .38 to .46; in the perpendicular position the index varies from 2.3 to 3.1, and the reflecting power from .41 to .34. For tellurium, in the parallel position the index varies from 1.9 to 2.9, the reflecting power from .10 to .27; in the perpendicular position the index varies from 1.7 to 2.7, the reflecting power from .09 to .23. The average error is estimated to be about 5 per cent. The accuracy is greatest for the middle range of wave-lengths, 3500–4000 A.

63. A suggested solar motor or thermal radiation recorder. RICHARD HAMER, University of Wisconsin.—It has been claimed that a homogeneous substance asymmetrically heated will give rise to thermo-electric currents. Actual experiments seem to support this, so that there should be little difficulty in making the following suggested motor work by intense thermal radiation. If the coil of a D'Arsonval galvanometer were replaced by a light sphere or cylinder of opaque material wound with copper wire in circular or oblong loops parallel to the axis placed normal to the field of the magnets, then marked heating by solar or other thermal radiation of one of the upper octants of the armature should produce a continuous rotation; for the thermo-electric current generated in one circuit of wire will give rise to a radial magnetic field that will tend to produce rotation, and this will in turn expose other sections to the radiation, making the motion continuous, another case of so-called perpetual motion, run by solar energy. Possibly its terminal speed of rotation could be used to measure and record the instantaneous intensity of any thermal or solar radiation.

64. Free and forced convection of heat in gases and liquids. II. CHESTER W. RICE, General Electric Research Laboratory.—Further work requires more general expressions for film thicknesses. For *free convection*, $B=KD(k/\mu c_p)^m[\nu/(\alpha g\Delta t)^{\frac{1}{3}}D^{\frac{2}{3}}]^n$, where K, m and n are constants, D characteristic dimension, k conductivity, μ viscosity, c_p specific heat, ν kinematic viscosity, α coefficient of expansion, g gravity, Δt temperature difference. Fluid properties are taken at the average temperature. For horizontal cylinders $K=2.12$, $m=1/4$ and $n=1/2$. Conductivities cannot be calculated from convection tests, unless their approximate values are known. Recalculations of k_0 in watt $cm^{-1}(°C)^{-1}$, give .0017, .0015, .0033, .0020, .0012, .0016, respectively, for transil oil, toluene, glycerine, analine, CCl_4 and olive oil. The temperature coefficients are −.0026, −.0029, −.0038, −.0018, −.0038 and + .0055. In *forced convection* $B=KD(k/\mu c_p)^m(\mu/\rho Dv)^n$, where ρ is density and v fluid velocity. For thin films and moderate Δt the convection becomes: $Wc=(Ak\Delta t/KD)\ (\mu/c_pk)^m(\rho Dv/\mu)^n$ watts, where A = area. For smooth pipes, well above the critical velocity, $m=1/2$, $n=5/6$ and $K=60$ approximately. A *relation between mechanical resistance and convection is* $Wc=(Ak\Delta tR/v\mu)$

$(\mu c_p/k)^n$ watts, where R = mechanical resistance, or $Wc = (Ac_p\Delta t R/v)\ (k/\mu c_p)^{1-n}$. Taking Lee's equation for R, $Wc = (k/\mu c_p)^{\frac{1}{3}} A\rho c_p\ v\Delta t\ [.0009 + .0763/(vD\rho/\mu)^{.25}]$ watts.

65. The thermal conductivities of alloys. ALPHEUS W. SMITH, Ohio State University.—The thermal and electrical conductivities of eighteen complete series of binary alloys have been measured. These series are: aluminum-antimony, aluminum-copper, aluminum-tin, aluminum-zinc, antimony-lead, antimony-tin, cadmium-bismuth, cadmium-tin, cadmium-zinc, manganese-nickel, manganese-copper, silver-copper, silver-lead, silver-tin, thallium-cadmium, thallium-lead, thallium-tin and tin-lead. The curves showing the thermal conductivities of a series of alloys as a function of the concentration of one of the components are similar in form to the corresponding curves showing the electrical conductivities as a function of the composition. In these alloys, however, the Wiedemann-Franz law holds only approximately. The presence of intermetallic compounds is indicated by sudden changes in the slope of the curves for both electrical and thermal conductivites. In the cases in which the alloys are solid solutions and also in the cases in which they are mechanical mixtures both the electrical and thermal conductivities show the characteristic behavior of these types of alloys.

66. Air flow through tubes. T. S. TAYLOR, Westinghouse Elec. and Mfg. Co.—A detailed study has been made of the changes in static pressure along a one inch smooth brass tube, 10 feet long, for various quantities of air, up to 20 cubic feet per minute, flowing through the tube. Special attention has been given to the changes in static pressure near the inlet and outlet of the tube. The quantity of air flowing was determined by making traverses of a second one inch tube 16 feet long, placed in series with the one being studied, by means of a small Pitot tube made from hypodermic needle material. The orifice constant has also been determined for three different orifices, one inch, $1\frac{1}{4}$ inch, and $1\frac{1}{2}$ inch in diameter, respectively. The orifices were made in the entrance to a large reservoir which joined directly to the tube being studied. From the data obtained, the coefficients of friction have also been calculated for the conditions under test.

67. An equation for head resistance of aircraft radiators. S. R. PARSONS, University of Arkansas.—The following equation is developed for the estimation of head resistance of certain types of radiators for aircraft engines: $R = kb\rho S^{1.95}[1 - 10^{-0.008x/r}]$ where R = head resistance in units of force per unit frontal area of radiator core; ρ = density of air; S = speed of air stream past the radiator; x = length of air tubes; r = hydraulic radius of air tubes; b = fraction of the frontal area of the core that is obstructed, or unity minus the fraction of the frontal area that is open for the passage of air; and k = a constant, dimensionless, if absolute units are used. The equation is in very fair agreement with actual measurements, for radiators of the "cellular" type, having circular or hexagonal air tubes with fairly smooth sides. It does not apply in any case where projections, holes, or other irregularities in the tube walls add to the resistance.

68. The nature of the constant of mass-action. R. D. KLEEMAN, Union College.—Van't Hoff has shown by thermodynamic reasoning that the constant of mass-action is a function of the temperature only, assuming that the molecules formed in the reacting chamber in his process can be removed so rapidly that no dissociation takes place. But the assumption is thermodynamically inadmissible, since it cannot be carried out without setting up finite heat gradients. If however the process is carried out infinitely slowly so that we are dealing with molecules in equilibrium with their dissociation products no thermodynamical objections arise. It can then be shown similarly that the constant of mass-action is strictly a function of the volume of the mixture, the masses of the constituents, and of the temperature. The same result may also be obtained by expressing in the equation of mass-action the partial concentrations or pressures of the

various molecules in the mixture in terms of the pressure of the mixture, and expanding it in a series of powers of the concentration of the constituents. This result is even obtained if the series is given a form corresponding to atoms not surrounded by fields of force. The existence of catalytic action by contact can also be deduced.

69. The simultaneous action on a hydrogen atom of crossed homogeneous electric and magnetic fields. OSCAR KLEIN, University of Michigan (Introduced by H. M. Randall).—By using a method due to Bohr, a direct calculation of the character of the motion and a determination of the stationary states of the hydrogen atom under the simultaneous influence of crossed homogeneous electric and magnetic fields has been made. Thus the motion is shown to be of a strictly doubly periodic character when powers of the fields higher than the first are neglected. This agrees with statements of Epstein and Halpert who did not, however, obtain a closed solution.

70. The specific heat of an elastic gyroscopic model of the hydrogen molecule. J. H. VAN VLECK, University of Minnesota.—A quantum theory interpretation of the specific heat of hydrogen on the basis of a non-gyroscopic elastic model was recently published by E. C. Kemble and the writer (Phys. Rev., **21**, 653, 1923). A vibrational degree of freedom (in addition to rotational effects previously considered by others) was used to account for the abnormally large specific heat at high temperatures. However, from general dynamical considerations Born has suggested that the hydrogen molecule possesses one quantum unit of electronic angular momentum and so is the natural analogue of the helium atom with axial symmetry proposed by Kemble and later independently by Bohr (M. Born, Die Naturwissenschaften, **10**, 677). A quantum theory of the specific heat of an elastic gyroscopic molecule has therefore been developed. A particular consequence is that, even when the vibrational degree of freedom is considered, the Born model of the hydrogen molecule has almost exactly the same specific heat as the non-gyroscopic model previously considered if in each case the probability is taken proportional to the total angular momentum. The Born model is therefore in satisfactory agreement with observed specific heats up to 2000°K provided the moment of inertia and vibrational frequency be 1.98×10^{-41} gm cm^2 and 1.46×10^{14} per sec.

71. Atomic force fields. F. S. BRACKETT, University of California.—Relations between the force acting on an electron and its distance from the nucleus are obtained from the energies associated with the circular orbits, these energies being known from spectroscopic data. In this determination the effect of external electronic charge is considered as well as the internal screening. From these relations simple expressions are found for the screening constants. The variation of the screening constant with increasing atomic number is related to a characteristic number which undergoes integral changes with the addition of each circular orbital group. This number is a fundamental constant of atomic development. Its changes occur whenever a "saturated condition" is reached and this condition is defined by a simple equation. The changes which have been observed in the characteristic rate of penetration of the virtual orbits (Brackett and Birge, Phys. Rev. **21**, 710, 1923) are coincident with the change in this number and are quantitatively related to it. This furnishes a strong physical basis for the quantum assignments of Brackett and Birge in the case of the deeply penetrating virtual orbits. The curves of this article also form the basis for further discussion of the questionable 3_3 assignments.

72. Atomic stability as tested photographically. WILLIAM D. HARKINS and R. W. RYAN, University of Chicago.—Forty thousand photographs of alpha ray tracks through air and argon have been taken by the use of a modified Shimizu-Wilson apparatus. These exhibit about eight very sharp collisions with atomic nuclei together with a considerable number of glancing impacts. The alpha rays were the fast rays from thorium C and C′. The nucleus of an argon atom remained intact when struck by an

alpha particle moving at the rate of fifteen thousand miles per second. In an ordinary collision three lines meet in a point. If the bombarded nucleus were to disintegrate this number would be increased to four. A photograph in which four lines meet in a point has been secured, but it does not exhibit conservation of momentum. In addition a double collision has been photographed. In this a bombarded argon nucleus almost immediately strikes a second argon nucleus and the tracks of both are visible. A new type of electron track has been observed.

73. The dependence of the loudness of a complex sound upon the energy in the various frequency regions of the sound. H. FLETCHER and J. C. STEINBERG, Research Laboratories of the American Telephone and Telegraph Co. and Western Electric Co., Inc.—By means of filters known amounts of energy were removed from various frequency ranges of a complex sound. The decrease in loudness due to this removal was measured by special methods described in the paper. This decrease was found to depend not only upon the relative intensities of these frequency regions but also upon the absolute values. At low intensities the contribution of a frequency region to loudness is proportional to the energy contained in the frequency region weighted according to the reciprocal of the minimum audible energy for that frequency. The total loudness may be obtained by summing the contributions throughout the frequency range of the sound. At the higher intensities, however, such a summation does not correspond even approximately with the observed results. The lack of agreement is due to the non-linear character of the transmitting mechanism of the ear. At these higher intensities it has been found possible to find the total loudness by summing a root of the weighted energy, the root and weighting factors depending upon the intensity and distribution of the sound. Examples of the methods of making loudness computations are given.

74. Preliminary analysis of four semi-vowel sounds. I. B. CRANDALL, Research Laboratories of the American Telephone and Telegraph Co. and the Western Electric Co., Inc.—The wave forms of four sounds, *l*, *m*, *n*, *ng*, as spoken six times each by two male voices have been recorded by apparatus which was nearly free from distortion in frequency and phase over the interval from 80 to 5000 cycles. A sample record so obtained was shown. The records of each sound have been analysed by the photo-mechanical method described in the following paper, and the results emphasize the vowel-like character of these sounds to such a degree that they deserve especial attention. These sounds were recorded as a by-product of an investigation of the standard vowel sounds in which eight voices were used, but in spite of the limited amount of data on the four semi-vowel sounds it is possible to give certain conclusions regarding their nature. The remarkable similarity between *l* and *e* is evident from one of the records shown; and the frequency analyses show that the four semi-vowel sounds all have double regions of resonance, which definitely relate them to each other and to the standard vowel diagram, so that they follow the sound *e* in the vowel triangle. The exact order of the three semi-vowel sounds following *l* is not easy to establish until more voices have been investigated, but the present data indicate tentatively the order *l*, *n*, *ng*, *m*, with the characteristics of *n* and *ng* so similar as to be almost indistinguishable.

75. Photo-mechanical analysis of vowel sounds. C. F. SACIA, Research Laboratories of the American Telephone and Telegraph Co. and the Western Electric Co., Inc. —The original oscillogram of the wave form was first transformed into a black profile record and joined to form an endless belt, which was passed at a definite speed repeatedly in front of a transverse slit. The fluctuations thus produced in a transmitted beam of light actuated a selenium cell which was connected across a tuned circuit to an amplifier and measuring instruments. By suitable adjustment of the belt speed and of the tuned circuit any frequency component of the original record could be measured by the

apparatus. Since the analysed wave is not strictly a periodic function, its frequency distribution is represented by a Fourier integral, not by a Fourier series. Periodic repetition of the entire wave, however, builds up a periodic function which is represented by a Fourier series whose fundamental period is the time duration of the wave. The frequency-amplitude diagram of this series thus consists of a dense arrangement of components spaced at small frequency intervals corresponding to the period of repetition. The envelope of this diagram approximates the frequency amplitude distribution of the wave analysed. By adjusting the period of repetition, all points in the frequency spectrum can be covered, if desired. We thus obtain a complete "inharmonic" or Fourier integral analysis of the wave.

76. On the human ear as a judge of small changes in the intensity of sound. HAWLEY O. TAYLOR, Boston, Mass.—The sounder consisted of a metallic plate which a hammer struck when released by a trigger. The sound measurer consisted of two telephone receivers attached back to back, and conductively connected to a 3-stage amplifier, the output of which passed through a transformer to a rectifier and galvanometer. The tests were made in the Supreme Court Chamber in the Capitol at Washington, D. C., in connection with its acoustical adjustment by the Mazer Acoustile Company. One-inch hair felt of known sound absorbing value was secured in pre-built wooden frames over which canvas was stretched one-eighth inch from the felt surface. These panels were attached to the walls of the Chamber, and the canvas painted with a flat, washable, oil paint. The person under test held the receivers, and, after being told the galvanometer readings corresponding to sounds heard, estimated the readings for a number of other sounds. The average errors in judging intensity changes for three persons tested was 54 per cent. Calling a decrease in intensity an increase was not infrequent. The human ear was thus found quite unreliable in the tests made.

77. Concerning the sound pressure-frequency characteristic of an electrical loud speaker with large diaphragm actuated by a force distributed approximately uniformly over its surface. C. W. HEWLETT, Research Laboratory, General Electric Co.—The author (Phys. Rev., **17**, 257, 1921; **19**, 52, 1922; Jour. Opt. Soc. Am., **4**, 1059, 1922) has discussed the theory and operation of an electrically driven diaphragm that is actuated by a force distributed over its surface. The loud speaker discussed in this abstract is the outgrowth of further work along the same line. To calculate the sound pressure-frequency characteristic for diaphragms of different sizes and masses, a number of assumptions had to be made, some of which hold more closely than others. They are as follows: (1) The induced current in the diaphragm is uniformly distributed; (2) The radial magnetic field is uniform; (3) The power dissipated in the diaphragm is independent of the frequency, and the same for all sizes of diaphragm; (4) The diaphragm is bounded by an infinite plane. The conclusions from the analysis are: (1) For low frequencies the sound pressure increases rapidly with increase of size of the diaphragm while for high frequencies, the sound pressure is independent of the size of the diaphragm. (2) There is a range of low frequencies over which the sound pressure is approximately independent of the frequency. This range gets narrower as the diaphragm gets larger. (3) The mass effect of the air surrounding such a diaphragm is a very important factor in determining the sound pressure. (4) The partial failure of assumptions (3) and (4) result in a longer range of the region over which the sound pressure is independent of the frequency.

78. The howling telephone and its application to bridge methods. LINDLEY PYLE, Washington University.—Oscillograms were presented showing the form of the currents in the transmitter and receiver circuits. Although the alternating receiver-current departs widely from the purely sinusoidal form, experiment shows that in bridge methods of measurement of inductance, capacitance and frequency this type of current yields, to

a high degree of precision, the same results as the sinusoidal type. With proper operating precautions steady oscillations may be maintained over extended intervals of time.

79. Causes of and remedies for the inefficiency of locomotive whistles. ARTHUR L. FOLEY, Indiana University.—Sound intensity measurements in different directions from a locomotive show that the locomotive whistle is in the wrong location to be most efficient. Reflection from the steam dome, smoke stack, etc., reflect the sound, the hot gases issuing from the smoke stack act as a dispersing lens, and, spreading back over the locomotive, reflect, refract and absorb considerable sound energy. Actual measurements in the case of one locomotive gave a sound intensity at right angles to the track of more than double the intensity in front of the locomotive. The writer, by placing the whistle inside of a specially designed reflector, has shown that the intensity of sound along the track in front of the locomotive can be increased several fold over that at right angles to the track. Proper proportioning the reflector and the whistle brought about also a considerable increase in sound intensity, due to the resonance of the reflector.

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